The Weekly Rigor

No. 169 “A mathematician is a machine for turning coffee into theorems.” September 16, 2017

101 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 5)

SET 1 SOLUTIONS

1. $f'(x) = 4(x^2+3)^3 2x = 8x(x^2+3)^3$.

2. $f'(x) = \frac{3}{2}(x^2+2)^{\frac{1}{2}} 2x = 3x(x^2+2)^{\frac{1}{2}} = 3x\sqrt{x^2+2}$.

3. $f'(x) = 5(x^3+x^2+2)^4(3x^2+2x) = 5(x^3+x^2+2)^4 x(3x+2) =$
$= 5x(x^3+x^2+2)^4(3x+2)$.

4. $f'(x) = \frac{5}{3}(x^3+x^2+4)^{\frac{2}{3}}(3x^2+2x) = \frac{5}{3}(x^3+x^2+4)^{\frac{2}{3}} x(3x+2) =$
$= \frac{5x}{3}(3x+2)(x^3+x^2+4)^{\frac{2}{3}} = \frac{5x}{3}(3x+2)\sqrt[3]{(x^3+x^2+4)^2}$.

5. $f'(x) = -3(x^2+2)^{-4} 2x = -6x(x^2+2)^{-4} = \frac{-6x}{(x^2+2)^4}$.

6. $f'(x) = -\frac{1}{2}(x^3+x^2+1)^{-\frac{3}{2}}(3x^2+2x) = -\frac{1}{2}(x^3+x^2+1)^{-\frac{3}{2}} x(3x+2) =$
$= \frac{-x(3x+2)}{2(x^3+x^2+1)^{\frac{3}{2}}} = \frac{-x(3x+2)}{2\sqrt{(x^3+x^2+1)^3}}$.

7. $f(x) = \frac{1}{(x^5+x^2)^3} = (x^5+x^2)^{-3} \Rightarrow f'(x) = -3(x^5+x^2)^{-4}(5x^4+2x) =$
$= -3(x^5+x^2)^{-4} x(5x^3+2) = \frac{-3x(5x^3+2)}{(x^5+x^2)^4}$.

Remark: I highly recommend that if the quotient rule can be easily avoided and turned into a power rule, as in this problem, you convert the original function.

8. $f(x) = \frac{1}{(x^4+x)^{\frac{5}{6}}} = (x^4+x)^{-\frac{5}{6}} \Rightarrow f'(x) = -\frac{5}{6}(x^4+x)^{-\frac{11}{6}}(4x^3+1) =$
$= \frac{-5(4x^3+1)}{6(x^4+x)^{\frac{11}{6}}} = \frac{-5(4x^3+1)}{6\sqrt[6]{(x^4+x)^{11}}}$.

9. $f(x) = \frac{1}{(x^5+x^2)^{\frac{3}{7}}} = (x^5 + x^2)^{-\frac{3}{7}} \Rightarrow f'(x) = -\frac{3}{7}(x^5 + x^2)^{-\frac{10}{7}}(5x^4 + 2x) =$

$= -\frac{3}{7}(x^5 + x^2)^{-\frac{10}{7}}x(5x^3 + 2) = -\frac{3x}{7}(x^5 + x^2)^{-\frac{10}{7}}(5x^3 + 2) = \dfrac{-3x(5x^3 + 2)}{7(x^5 + x^2)^{\frac{10}{7}}} =$

$= \frac{-3x(5x^3+2)}{7\sqrt[7]{(x^5+x^2)^{10}}}.$

10. $f(x) = \sqrt{x^2 + 3} = (x^2 + 3)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(x^2 + 3)^{-\frac{1}{2}}2x = \frac{x}{(x^2+3)^{\frac{1}{2}}} = \frac{x}{\sqrt{x^2+3}}.$

Remark: If the original function uses a radical sign, always convert it to using the exponential notation before finding the derivative function.

11. $f(x) = \sqrt[4]{x^3 + x^2 + 4} = (x^3 + x^2 + 4)^{\frac{1}{4}} \Rightarrow f'(x) = \frac{1}{4}(x^3 + x^2 + 4)^{-\frac{3}{4}}(3x^2 + 2x) =$

$= \frac{1}{4}(x^3 + x^2 + 4)^{-\frac{3}{4}}x(3x + 2) = \frac{1}{4}x(x^3 + x^2 + 4)^{-\frac{3}{4}}(3x + 2) = \dfrac{x(3x + 2)}{4(x^3 + x^2 + 4)^{\frac{3}{4}}} =$

$= \frac{x(3x+2)}{4\sqrt[4]{(x^3+x^2+4)^3}}.$

12. $f(x) = \sqrt[3]{(x^2 + x)^2} = ((x^2 + x)^2)^{\frac{1}{3}} = (x^2 + x)^{\frac{2}{3}} \Rightarrow$

$\Rightarrow f'(x) = \frac{2}{3}(x^2 + x)^{-\frac{1}{3}}(2x + 1) = \frac{2(2x+1)}{3(x^2+x)^{\frac{1}{3}}} = \frac{2(2x+1)}{3\sqrt[3]{x^2+x}}.$

13. $f(x) = \left(\sqrt{x^3 + 4}\right)^3 = \left((x^3 + 4)^{\frac{1}{2}}\right)^3 = (x^3 + 4)^{\frac{3}{2}} \Rightarrow f'(x) = \frac{3}{2}(x^3 + 4)^{\frac{1}{2}}3x^2 =$

$= \frac{9}{2}x^2\sqrt{x^3 + 4}.$

14. $f(x) = \frac{1}{\sqrt[3]{x-1}} = \frac{1}{(x-1)^{\frac{1}{3}}} = (x - 1)^{-\frac{1}{3}} \Rightarrow f'(x) = -\frac{1}{3}(x - 1)^{-\frac{4}{3}} = \frac{-1}{3(x-1)^{\frac{4}{3}}} = \frac{-1}{3\sqrt[3]{(x-1)^4}}.$

15. $f(x) = \frac{1}{\sqrt{(x^3+x)^5}} = \frac{1}{(x^3+x)^{\frac{5}{2}}} = (x^3 + x)^{-\frac{5}{2}} \Rightarrow f'(x) = -\frac{5}{2}(x^3 + x)^{-\frac{7}{2}}(3x^2 + 1) =$

$= \frac{-5(3x^2+1)}{2(x^3+x)^{\frac{7}{2}}} = \frac{-5(3x^2+1)}{2\sqrt{(x^3+x)^7}}.$

16. $f'(x) = 3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{2x(x+1)-(x^2+3)}{(x+1)^2}\right) = 3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{2x^2+2x-x^2-3}{(x+1)^2}\right) = 3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{x^2+2x-3}{(x+1)^2}\right) =$

$= 3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{(x+3)(x-1)}{(x+1)^2}\right).$