The Weekly Rigor

No. 170 "A mathematician is a machine for turning coffee into theorems." September 23, 2017

101 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 6)

17. $f'(x)=4\left(\frac{x-3}{x+8}\right)^3\left(\frac{(x+8)-(x-3)}{(x+8)^2}\right)=4\left(\frac{x-3}{x+8}\right)^3\left(\frac{x+8-x+3}{(x+8)^2}\right)=4\left(\frac{x-3}{x+8}\right)^3\left(\frac{11}{(x+8)^2}\right)=44\left(\frac{x-3}{x+8}\right)^3.$

18. $f'(x)=5\left(\frac{x^2-4}{x^3+7}\right)^4\left(\frac{2x(x^3+7)-(x^2-4)3x^2}{(x^3+7)^2}\right)=5\left(\frac{x^2-4}{x^3+7}\right)^4\left(\frac{2x^4+14x-3x^4+12x^2}{(x^3+7)^2}\right)=$
$=5\left(\frac{x^2-4}{x^3+7}\right)^4\left(\frac{-x^4+14x+12x^2}{(x^3+7)^2}\right).$

19. $f'(x)=3\left(\frac{x^4-x^5}{x^2+x^3}\right)^2\left(\frac{(4x^3-5x^4)(x^2+x^3)-(x^4-x^5)(2x+3x^2)}{(x^2+x^3)^2}\right)=$
$=3\left(\frac{x^4-x^5}{x^2+x^3}\right)^2\left(\frac{x^3(4-5x)x^2(1+x)-x^4(1-x)x(2+3x)}{(x^2+x^3)^2}\right)=$
$=3\left(\frac{x^4-x^5}{x^2+x^3}\right)^2\left(\frac{x^5(4-5x)(1+x)-x^5(1-x)(2+3x)}{(x^2+x^3)^2}\right).$

20. $f'(x)=3\left(x^{\frac{2}{3}}+x^{\frac{1}{2}}\right)^2\left(\frac{2}{3}x^{-\frac{1}{3}}+\frac{1}{2}x^{-\frac{1}{2}}\right).$

21. $f(x)=\left(\sqrt{x}+\sqrt[3]{x^2}\right)^{\frac{4}{3}}=\left(x^{\frac{1}{2}}+x^{\frac{2}{3}}\right)^{\frac{4}{3}}\Rightarrow f'(x)=\frac{4}{3}\left(x^{\frac{1}{2}}+x^{\frac{2}{3}}\right)^{\frac{1}{3}}\left(\frac{1}{2}x^{-\frac{1}{2}}+\frac{2}{3}x^{-\frac{1}{3}}\right).$

22. $f(x)=\left(\sqrt[3]{x^5}+\sqrt[5]{x^{\frac{9}{2}}}\right)^{\frac{4}{3}}=\left((x^5)^{\frac{1}{3}}+\left(x^{\frac{9}{2}}\right)^{\frac{1}{5}}\right)^{\frac{4}{3}}=\left(x^{\frac{5}{3}}+x^{\frac{9}{10}}\right)^{\frac{4}{3}}\Rightarrow$
$\Rightarrow f'(x)=\frac{4}{3}\left(x^{\frac{5}{3}}+x^{\frac{9}{10}}\right)^{\frac{1}{3}}\left(\frac{5}{3}x^{\frac{2}{3}}+\frac{9}{10}x^{-\frac{1}{10}}\right).$

23. $f(x)=\sqrt[5]{1+x^{\frac{2}{3}}}=\left(1+x^{\frac{2}{3}}\right)^{\frac{1}{5}}\Rightarrow f'(x)=\frac{1}{5}\left(1+x^{\frac{2}{3}}\right)^{-\frac{4}{5}}\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{15}x^{-\frac{1}{3}}\left(1+x^{\frac{2}{3}}\right)^{-\frac{4}{5}}.$

24. $f(x)=\sqrt[4]{\sqrt{x}+x^{\frac{4}{3}}}=\left(x^{\frac{1}{2}}+x^{\frac{4}{3}}\right)^{\frac{1}{4}}\Rightarrow f'(x)=\frac{1}{4}\left(x^{\frac{1}{2}}+x^{\frac{4}{3}}\right)^{-\frac{3}{4}}\left(\frac{1}{2}x^{-\frac{1}{2}}+\frac{4}{3}x^{\frac{1}{3}}\right).$

25. $f'(x)=-4\left(x^{\frac{3}{4}}+x^{\frac{1}{2}}\right)^{-5}\left(\frac{3}{4}x^{-\frac{1}{4}}+\frac{1}{2}x^{-\frac{1}{2}}\right).$

26. $f(x)=\frac{1}{\left(\sqrt[3]{x}+x^2\right)^{-2}}=\left(x^{\frac{1}{3}}+x^2\right)^2 \Rightarrow f'(x)=2\left(x^{\frac{1}{3}}+x^2\right)\left(\frac{1}{3}x^{-\frac{2}{3}}+2x\right)$.

27. $f(x)=\frac{1}{\sqrt[4]{\sqrt{x}+x}}=\frac{1}{\left(x^{\frac{1}{2}}+x\right)^{\frac{1}{4}}}=\left(x^{\frac{1}{2}}+x\right)^{-\frac{1}{4}} \Rightarrow f'(x)=-\frac{1}{4}\left(x^{\frac{1}{2}}+x\right)^{-\frac{5}{4}}\left(\frac{1}{2}x^{-\frac{1}{2}}+1\right)$.

28. $f(x)=\frac{1}{\sqrt[5]{\left(\sqrt{x}+x^{\frac{1}{3}}\right)^2}}=\frac{1}{\left(\left(x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^2\right)^{\frac{1}{5}}}=\frac{1}{\left(x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^{\frac{2}{5}}}=\left(x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^{-\frac{2}{5}} \Rightarrow$

$\Rightarrow f'(x)=-\frac{2}{5}\left(x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^{-\frac{7}{5}}\left(\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{3}x^{-\frac{2}{3}}\right)$.

29. $f'(x)=e^{3x}(3)=3e^{3x}$.

30. $f'(x)=e^{\frac{2}{5}x}\left(\frac{2}{5}\right)=\frac{2}{5}e^{\frac{2}{5}x}$.

31. $f'(x)=e^{x^2}(2x)=2xe^{x^2}$.

32. $f'(x)=e^{x^3}(3x^2)=3x^2e^{x^3}$.

33. $f'(x)=e^{e^x}(e^x)=e^xe^{e^x}=e^{x+e^x}$.

34. $f'(x)=e^{e^{2x}}(e^{2x})(2)=2e^{2x}e^{e^{2x}}=2e^{2x+e^{2x}}$.

35. $f'(x)=e^{\ln(x)}\left(\frac{1}{x}\right)=\frac{e^{\ln(x)}}{x}=\frac{x}{x}=1$.

36. $f'(x)=e^{\ln(x^2)}\left(\frac{1}{x^2}\right)(2x)=\frac{2e^{\ln(x^2)}}{x}=\frac{2x^2}{x}=2x$.

37. $f'(x)=e^{\arctan(x)}\left(\frac{1}{1+x^2}\right)=\frac{e^{\arctan(x)}}{1+x^2}$.

38. $f'(x)=e^{\arcsin(2x)}\left(\frac{1}{\sqrt{1+(2x)^2}}\right)(2)=\frac{2e^{\arcsin(2x)}}{\sqrt{1+4x^2}}$.