The Weekly Rigor

No. 171   "A mathematician is a machine for turning coffee into theorems."   September 30, 2017

101 Problems in Calculating Derivatives Using the Chain Rule with Solutions
(Part 7)

39. $f'(x) = 3(e^x + e^{-x})^2(e^x + e^{-x}(-1)) = 3(e^x + e^{-x})^2(e^x - e^{-x}).$

40. $f(x) = \frac{1}{\sqrt{e^{2x} - e^{3x}}} = \frac{1}{(e^{2x} - e^{3x})^{\frac{1}{2}}} = (e^{2x} - e^{3x})^{-\frac{1}{2}} \Rightarrow$

$f'(x) = -\frac{1}{2}(e^{2x} - e^{3x})^{-\frac{3}{2}}(e^{2x}(2) - e^{3x}(3)) = -\frac{1}{2}(e^{2x} - e^{3x})^{-\frac{3}{2}}(2e^{2x} - 3e^{3x}) =$

$= -\frac{1}{2}e^{2x}(2 - 3e^x)(e^{2x} - e^{3x})^{-\frac{3}{2}}.$

41. $f'(x) = \frac{1}{2x}(2) = \frac{2}{2x} = \frac{1}{x}.$
     Alternate Solution:
Using the logarithmic properties,
$f(x) = \ln(2x) = \ln(2) + \ln(x) \Rightarrow f'(x) = 0 + \frac{1}{x} = \frac{1}{x}.$

42. $f'(x) = \frac{1}{3x}(3) = \frac{3}{3x} = \frac{1}{x}.$
     Alternate Solution:
Using the logarithmic properties,
$f(x) = \ln(3x) = \ln(3) + \ln(x) \Rightarrow f'(x) = 0 + \frac{1}{x} = \frac{1}{x}.$

43. $f'(x) = \frac{1}{x^2}(2x) = \frac{2x}{x^2} = \frac{2}{x}.$
     Alternate Solution:
Using the logarithmic properties,
$f(x) = \ln(x^2) = 2\ln(x) \Rightarrow f'(x) = 2\left(\frac{1}{x}\right) = \frac{2}{x}.$

44. $f'(x) = \frac{1}{3x^2}(6x) = \frac{6x}{3x^2} = \frac{2}{x}$.

Alternate Solution:

Using the logarithmic properties,

$f(x) = \ln(3x^2) = \ln(3) + \ln(x^2) = \ln(3) + 2\ln(x) \Rightarrow f'(x) = 0 + 2\left(\frac{1}{x}\right) = \frac{2}{x}$.

45. $f(x) = \ln^2(x) = [\ln(x)]^2 \Rightarrow f'(x) = 2\ln(x)\frac{1}{x} = \frac{2\ln(x)}{x}$.

Remark: Converting the original function as I have done here clarifies what the "outside" function is as opposed to the "inside" function.

46. $f(x) = \ln^3(5x) = [\ln(5x)]^3 \Rightarrow f'(x) = 3[\ln(5x)]^2\frac{1}{5x}(5) = 3\ln^2(5x)\frac{1}{5x}(5) = \frac{3\ln^2(5x)}{x}$.

47. $f(x) = \ln^4(3x^5) = [\ln(3x^5)]^4 \Rightarrow f'(x) = 4\ln^3(3x^5)\frac{1}{3x^5}(15x^4) = \frac{60x^4\ln^3(3x^5)}{3x^5} =$

$= \frac{20\ln^3(3x^5)}{x}$.

48. $f'(x) = \frac{1}{e^x}(e^x) = \frac{e^x}{e^x} = 1$.

Alternate Solution:

Using the logarithmic properties,

$f(x) = \ln(e^x) = x\ln(e) = x(1) = x \Rightarrow f'(x) = 1$.

49. $f'(x) = \frac{1}{e^{x^2}}\left(e^{x^2}\right)(2x) = \frac{2xe^{x^2}}{e^{x^2}} = 2x$.

Alternate Solution:

Using the logarithmic properties,

$f(x) = \ln\left(e^{x^2}\right) = x^2\ln(e) = x^2(1) = x^2 \Rightarrow f'(x) = 2x$.

50. $f'(x) = \frac{1}{\ln(x)}\left(\frac{1}{x}\right) = \frac{1}{x\ln(x)}$.

51. $f'(x) = \frac{1}{\ln(2x)}\left(\frac{1}{2x}\right)(2) = \frac{1}{x\ln(x)}$.