The Weekly Rigor

No. 172 "A mathematician is a machine for turning coffee into theorems." October 7, 2017

101 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 8)

52. $f'(x) = \frac{1}{\arctan(x)}\left(\frac{1}{1+x^2}\right) = \frac{1}{(1+x^2)\arctan(x)}$.

53. $f'(x) = \frac{1}{\arcsin(x)}\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}\arcsin(x)}$.

54. $f'(x) = 3(\ln(2x) + \ln(x))^2\left(\frac{2}{2x} + \frac{1}{x}\right) = 3(\ln(2x) + \ln(x))^2\left(\frac{1}{x} + \frac{1}{x}\right) =$
$= 3(\ln(2x) + \ln(x))^2\left(\frac{2}{x}\right) = \frac{6(\ln(2x)+\ln(x))^2}{x}$.

55. $f'(x) = -2\left(\ln^2(x^5) - \ln(x)\right)^{-3}\left(2\ln(x^5)\frac{5x^4}{x^5} + \frac{1}{x}\right) =$
$= -2\left(\ln^2(x^5) - \ln(x)\right)^{-3}\left(\frac{10}{x}\ln(x^5) + \frac{1}{x}\right) = -2\left(\ln^2(x^5) - \ln(x)\right)^{-3}\left(\frac{10\ln(x^5)+1}{x}\right)$.

56. $f'(x) = \cos(3x)(3) = 3\cos(3x)$.

57. $f'(x) = -\sin\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right) = -\frac{1}{2}\sin\left(\frac{1}{2}x\right)$.

58. $f'(x) = \sec^2(3x)(3) = 3\sec^2(3x)$.

59. $f'(x) = \cos(x^2)(2x) = 2x\cos(x^2)$.

60. $f'(x) = -\sin\left(x^{\frac{3}{4}}\right)\frac{3}{4}x^{\frac{-1}{4}} = -\frac{3}{4}x^{\frac{-1}{4}}\sin\left(x^{\frac{3}{4}}\right)$.

61. $f'(x) = \sec^2(x^5)5x^4 = 5x^4\sec^2(x^5)$.

62. $f(x) = \sin^2(x) = [\sin(x)]^2 \Rightarrow f'(x) = 2\sin(x)\cos(x)$.

Remark: The conversion of the original function, as in problems 45-47, is done to distinguish the "outside" from the "inside" function.

63. $f(x) = \cos^3(x) = [\cos(x)]^3 \Rightarrow f'(x) = 3[\cos(x)]^2(-\sin(x)) = -3\cos^2(x)\sin(x)$.

64. $f(x) = \tan^4(x) = [\tan(x)]^4 \Rightarrow f'(x) = 4\tan^3(x)\sec^2(x)$.

65. $f(x) = \sin^5(x^2) = [\sin(x^2)]^5 \Rightarrow f'(x) = 5\sin^4(x^2)\cos(x^2)2x =$
$= 10x\sin^4(x^2)\cos(x^2)$.

66. $f(x) = \cos^5(x^2) = [\cos(x^2)]^5 \Rightarrow f'(x) = -5\cos^4(x^2)\sin(x^2)2x =$
$= -10x\cos^4(x^2)\sin(x^2)$.

67. $f(x) = \tan^4(x^5) = [\tan(x^5)]^4 \Rightarrow f'(x) = 4\tan^3(x^5)\sec^2(x^5)5x^4 =$
$= 20x^4\tan^3(x^5)\sec^2(x^5)$.

68. $f'(x) = \cos(e^x)(e^x) = e^x\cos(e^x)$.

69. $f'(x) = \sec^2(e^{3x})(e^{3x})(3) = 3e^{3x}\sec^2(e^{3x})$.

70. $f'(x) = \frac{3}{1+(3x)^2} = \frac{3}{1+9x^2}$.

71. $f(x) = \arctan^2(x) = [\arctan(x)]^2 \Rightarrow f'(x) = 2\arctan(x)\left(\frac{1}{1+x^2}\right) = \frac{2\arctan(x)}{1+x^2}$.

72. $f(x) = \arctan^2(3x) = [\arctan(3x)]^2 \Rightarrow f'(x) = 2\arctan(3x)\left(\frac{3}{1+(3x)^2}\right) = \frac{6\arctan(x)}{1+9x^2}$.

73. $f(x) = \arcsin^3(x) = [\arcsin(x)]^3 \Rightarrow f'(x) = 3\arcsin^2(x)\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{3\arcsin^2(x)}{\sqrt{1-x^2}}$.

74. $f(x) = \arcsin^3(x^2) = [\arcsin(x^2)]^3 \Rightarrow f'(x) = 3\arcsin^2(x^2)\left(\frac{2x}{\sqrt{1-x^4}}\right) = \frac{6x\arcsin^2(x^2)}{\sqrt{1-x^4}}$.

75. $f'(x) = \frac{-1}{\sqrt{1-(e^x)^2}}(e^x) = \frac{-e^x}{\sqrt{1-e^{2x}}}$.