# The Weekly Rigor

No. 173 "A mathematician is a machine for turning coffee into theorems." October 14, 2017

## 101 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 9)

76. $f'(x) = \cos(\arctan(x))\left(\frac{1}{1+x^2}\right) = \frac{\cos(\arctan(x))}{1+x^2}$.

77. $f'(x) = \sec^2(\arcsin(x))\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{\sec^2(\arcsin(x))}{\sqrt{1-x^2}}$.

78. $f'(x) = \frac{1}{1+\sin^2(x)}(\cos(x)) = \frac{\cos(x)}{1+\sin^2(x)}$.

79. $f'(x) = \frac{1}{\sqrt{1-\tan^2(x)}}\left(\sec^2(x)\right) = \frac{\sec^2(x)}{\sqrt{1-\tan^2(x)}}$.

80. $f'(x) = \frac{1}{\sqrt{1-\arccos^2(x)}}\left(\frac{-1}{\sqrt{1-x^2}}\right)$.

81. $f'(x) = \frac{1}{1+\arcsin^2(x)}\left(\frac{1}{\sqrt{1-x^2}}\right)$.

82. $f'(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$.

83. $f(x) = \ln\left(1+\sqrt{x}\right) = \ln\left(1+x^{\frac{1}{2}}\right) \Rightarrow f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1+x^{\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}}\left(1+x^{\frac{1}{2}}\right)} = \frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)}$.

84. $f'(x) = \sec^2(\sin(x))\cos(x)$.

85. $f'(x) = 3(x^3+1)^2 3x^2(5+x^2)^4 + (x^3+1)^3 4(5+x^2)^3 2x =$
$= 9x^2(x^3+1)^2(5+x^2)^4 + 8x(x^3+1)^3(5+x^2)^3$.

86. $f'(x) = 4(x^2+3)^3 2x(x^2+2)^{\frac{3}{2}} + (x^2+3)^4 \frac{3}{2}(x^2+2)^{\frac{1}{2}} 2x =$
$= 8x(x^2+3)^3(x^2+2)^{\frac{3}{2}} + 3x(x^2+3)^4(x^2+2)^{\frac{1}{2}}.$

87. $f'(x) = 2(x^2+x)^1(2x+1)(-x^2+x^3)^{\frac{3}{2}} +$
$+(x^2+x)^2 \frac{3}{2}(-x^2+x^3)^{\frac{1}{2}}(-2x+3x)^2.$

88. $f(x) = \sqrt{x^3+1}(x^2+1)^4 = (x^3+1)^{\frac{1}{2}}(x^2+1)^4 \Rightarrow$
$\Rightarrow f'(x) = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)(x^2+1)^4 + (x^3+1)^{\frac{1}{2}} 4(x^2+1)^3(2x) =$
$= \frac{3}{2}x^2(x^3+1)^{-\frac{1}{2}}(x^2+1)^4 + 8x(x^3+1)^{\frac{1}{2}}(x^2+1)^3.$

89. $f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x).$

90. $f'(x) = 2e^{2x}\tan^3(x) + 3e^{2x}\tan^2(x)\sec^2(x) = e^{2x}\tan^2(x)\left[2\tan(x) + 3\sec^2(x)\right].$

91. $f'(x) = \frac{4(x^2+3)^3(2x)(x^2+2)^{\frac{3}{2}} + \frac{3}{2}(x^2+2)^{\frac{1}{2}}(2x)(x^2+3)^4}{(x^2+2)^3} = \frac{8x(x^2+3)^3(x^2+2)^{\frac{3}{2}} + 3x(x^2+2)^{\frac{1}{2}}(x^2+3)^4}{(x^2+2)^3}.$

92. $f'(x) = \frac{2e^{2x}\sin(3x) + 3\cos(3x)e^{2x}}{\sin^2(3x)}.$

93. $f'(x) = \frac{\frac{1}{4x}(4)\arctan(3x) - \ln(4x)\frac{1}{1+9x^2}(3)}{\arctan^2(3x)} = \frac{\frac{\arctan(3x)}{x} - \frac{3\ln(4x)}{1+9x^2}}{\arctan^2(3x)}.$