The Weekly Rigor

No. 174 "A mathematician is a machine for turning coffee into theorems." October 21, 2017

101 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 10)

94. $f'(x) = e^{\cos(4x)}(-\sin(4x))4 = -4\sin(4x)e^{\cos(4x)}$.

95. $f'(x) = e^{\tan(\ln(3x))}\sec^2(\ln(3x))\left(\frac{1}{3x}\right)(3) = \frac{e^{\tan(\ln(3x))}\sec^2(\ln(3x))}{x}$.

96. $f'(x) = 3\sin^2(\cos(2x))\cos(\cos(2x))(-\sin(2x))(2) =$
$= -6\sin(2x)\cos(\cos(2x))\sin^2(\cos(2x))$.

97. $f(x) = \tan^4\left(\ln\left(e^{\sin(3x)}\right)\right) = \left(\tan\left(\ln\left(e^{\sin(3x)}\right)\right)\right)^4 \Rightarrow$

$$f'(x) = 4\left(\tan\left(\ln\left(e^{\sin(3x)}\right)\right)\right)^3\sec^2\left(\ln\left(e^{\sin(3x)}\right)\right)\left(\frac{1}{e^{\sin(3x)}}\right)\left(e^{\sin(3x)}\right)(\cos(3x))(3) =$$

$$= 4\left(\tan\left(\ln\left(e^{\sin(3x)}\right)\right)\right)^3\sec^2\left(\ln\left(e^{\sin(3x)}\right)\right)\left(\frac{e^{\sin(3x)}\,3\cos(3x)}{e^{\sin(3x)}}\right) =$$

$$= 4\tan^3\left(\ln\left(e^{\sin(3x)}\right)\right)\sec^2\left(\ln\left(e^{\sin(3x)}\right)\right)\left(\frac{e^{\sin(3x)}\,3\cos(3x)}{e^{\sin(3x)}}\right) =$$

$$= 4\tan^3\left(\ln\left(e^{\sin(3x)}\right)\right)\sec^2\left(\ln\left(e^{\sin(3x)}\right)\right)3\cos(3x) = 12\cos(3x)\tan^3\left(\ln\left(e^{\sin(3x)}\right)\right)\sec^2\left(\ln\left(e^{\sin(3x)}\right)\right).$$

98. $f(x)=\ln^4\left(\cos\left(e^{\sin(x^2)}\right)\right)=\left(\ln\left(\cos\left(e^{\sin(x^2)}\right)\right)\right)^4 \Rightarrow$

$f'(x)=4\left(\ln\left(\cos\left(e^{\sin(x^2)}\right)\right)\right)^3\frac{1}{\cos\left(e^{\sin(x^2)}\right)}\left(-\sin\left(e^{\sin(x^2)}\right)\right)e^{\sin(x^2)}(\cos(x^2))(2x)=$

$$=\frac{-8xe^{\sin(x^2)}\cos(x^2)\sin\left(e^{\sin(x^2)}\right)\left(\ln\left(\cos\left(e^{\sin(x^2)}\right)\right)\right)^3}{\cos\left(e^{\sin(x^2)}\right)}=$$

$$=\frac{-8xe^{\sin(x^2)}\cos(x^2)\sin\left(e^{\sin(x^2)}\right)\ln^3\left(\cos\left(e^{\sin(x^2)}\right)\right)}{\cos\left(e^{\sin(x^2)}\right)}=$$

$$=-8xe^{\sin(x^2)}\cos(x^2)\tan\left(e^{\sin(x^2)}\right)\ln^3\left(\cos\left(e^{\sin(x^2)}\right)\right).$$

99. $f(x)=\arctan^4(\cos(\ln(5x)))=\left(\arctan(\cos(\ln(5x)))\right)^4 \Rightarrow$

$f'(x)=4\left(\arctan(\cos(\ln(5x)))\right)^3\frac{1}{1+\cos^2(\ln(5x))}(-\sin(\ln(5x)))\left(\frac{1}{5x}\right)(5)=$

$$=\frac{-20\sin(\ln(5x))\left(\arctan(\cos(\ln(5x)))\right)^3}{x\left(1+\cos^2(\ln(5x))\right)}=\frac{-20\sin(\ln(5x))\arctan^3(\cos(\ln(5x)))}{x\left(1+\cos^2(\ln(5x))\right)}\ .$$

100. $f(x)=\sin\left(\sin^2\left(\sin^3(x^4)\right)\right)=\sin\left(\left[\sin\left(\sin^3(x^4)\right)\right]^2\right)=\sin([\sin([\sin(x^4)]^3)]^2)\Rightarrow$

$f'(x)=$

$=\cos\left(\sin^2\left(\sin^3(x^4)\right)\right)\left(2\sin\left(\sin^3(x^4)\right)\right)\left(\cos\left(\sin^3(x^4)\right)\right)\left(3\sin^2(x^4)\right)(\cos(x^4))4x^3=$

$=24x^3\cos\left(\sin^2\left(\sin^3(x^4)\right)\right)\left(\sin\left(\sin^3(x^4)\right)\right)\left(\cos\left(\sin^3(x^4)\right)\right)\left(\sin^2(x^4)\right)(\cos(x^4)).$

101. $f(x)=\arctan\left(\sin\left(\ln\left(e^{\sqrt{x}}\right)\right)\right)=\arctan\left(\sin\left(\ln\left(e^{x^{\frac{1}{2}}}\right)\right)\right)\Rightarrow$

$f'(x)=\frac{1}{1+\sin^2\left(\ln\left(e^{x^{\frac{1}{2}}}\right)\right)}\left(\cos\left(\ln\left(e^{x^{\frac{1}{2}}}\right)\right)\right)\left(\frac{1}{e^{x^{\frac{1}{2}}}}\right)\left(e^{x^{\frac{1}{2}}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)=$

$$=\frac{\cos\left(\ln\left(e^{x^{\frac{1}{2}}}\right)\right)}{2x^{\frac{1}{2}}\left(1+\sin^2\left(\ln\left(e^{x^{\frac{1}{2}}}\right)\right)\right)}=\frac{\cos\left(\ln\left(e^{\sqrt{x}}\right)\right)}{2\sqrt{x}\left(1+\sin^2\left(\ln\left(e^{\sqrt{x}}\right)\right)\right)}\ .$$