The Weekly Rigor

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A Positive Times a Negative Equals a Negative: An Alternate Proof

This issue offers a proof that a positive number times a negative number equals a negative number, for any real numbers. This proof differs from the one demonstrated as Theorem 8 of *WR* no. 162. Reference to the real number axioms and the Fundamental Multiplication Property are found in *WR* no. 160. In the following, a and b denote real numbers.

Before proceeding to the abstract proof, let us see the reasoning using concrete integers. We claim, say, that $4(-5) = -(4 \cdot 5)$. We prove this claim as follows.

Claim: $$4(-5) = -(4 \cdot 5).$$

Proof: $$5 + (-5) = 0,$$
by Axiom 6. Hence,
$$4\big(5 + (-5)\big) = 4 \cdot 0,$$
by the Fundamental Multiplication Property. So,
$$4\big(5 + (-5)\big) = 0,$$
by Theorem 4 of *WR* no. 160. Thus,
$$4 \cdot 5 + 4(-5) = 0,$$
by Axiom 4. But
$$4 \cdot 5 + [-(4 \cdot 5)] = 0,$$
by Axiom 6. Consequently,
$$4 \cdot 5 + 4(-5) = 4 \cdot 5 + [-(4 \cdot 5)].$$
Therefore,
$$4(-5) = -(4 \cdot 5),$$
by Theorem 1 of *WR* no. 160.

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Remark: Without special comment, we shall include in this claim the variation
$$(-4)5 = -(4 \cdot 5).$$

And now we generalize from the particular numbers 4 and 5 to any real numbers a and b, using exactly the same steps of reasoning.

Theorem 1: $$a(-b) = -(ab).$$

Proof: $$b + (-b) = 0,$$
by Axiom 6. Hence,
$$a\big(b + (-b)\big) = a \cdot 0,$$
by the Fundamental Multiplication Property. So,
$$a\big(b + (-b)\big) = 0,$$
by Theorem 4 of *WR* no. 160. Thus,
$$ab + a(-b) = 0,$$
by Axiom 4. But
$$ab + [-(ab)] = 0,$$
by Axiom 6. Consequently,
$$ab + a(-b) = ab + [-(ab)].$$
Therefore,
$$a(-b) = -(ab),$$
by Theorem 1 of *WR* no. 160.

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Remark: Without special comment, we shall include in this theorem the variation
$$(-a)b = -(ab).$$