The Weekly Rigor

No. 192 "A mathematician is a machine for turning coffee into theorems." February 24, 2018

52 Problems in Calculating Composite Functions of Polynomials

(Part 3)

SELECTED SOLUTIONS

1. $f\big(g(x)\big) = f(x-3) = x-3.$

2. $f\big(g(x)\big) = f(x+1) = x+1.$

3. $f\big(g(x)\big) = f(x-3) = (x-3)+2 = x-3+2 = x-1.$

4. $f\big(g(x)\big) = f(x+1) = (x+1)+1 = x+1+1 = x+2.$

5. $f\big(g(x)\big) = f(x+1) = (x+1)^2 = (x+1)(x+1) = x^2+x+x+1 = x^2+2x+1.$

6. $f\big(g(x)\big) = f(x^2+3) = (x^2+3)^2 = x^4+6x^2+9.$

7. $f\big(g(x)\big) = f(3x-1) = 1-(3x-1)^2 = 1-[(3x)^2-6x+1] = 1-9x^2+6x-1 =$
$= -9x^2+6x.$

8. $f\big(g(x)\big) = f(x+1) = 3+(x+1)-(x+1)^2 = 3+x+1-[x^2+2x+1] =$
$= 4+x-[x^2+2x+1] = 4+x-x^2-2x-1 = 3-x-x^2 = -x^2-x+3.$

10. $f\big(g(x)\big) = f(1-x^2) = 3(1-x^2)-1 = 3-3x^2-1 = 2-3x^2 = -3x^2+2.$

11. $g\big(f(x)\big) = g(x+2) = (x+2)-3 = x+2-3 = x-1.$

12. $g\big(f(x)\big) = g(x^2) = x^2+1.$

13. $g\big(f(x)\big) = g(x^2) = (x^2)^2+3 = x^4+3.$

15. $g\big(f(x)\big) = g(3+x-x^2) = (3+x-x^2)+1 = 3+x-x^2+1 = 4+x-x^2 =$
$= -x^2+x+4.$

17. $g\big(f(x)\big) = g(3x-1) = 1-(3x-1)^2 = 1-[9x^2-6x+1] = 1-9x^2+6x-1 =$
$= -9x^2+6x.$

18. $f\big(f(x)\big) = f(x) = x.$

19. $f\big(f(x)\big) = f(x^2) = (x^2)^2 = x^4.$

21. $f\big(f(x)\big) = f(3 + x - x^2) = 3 + (3 + x - x^2) - (3 + x - x^2)^2 =$
$= 3 + 3 + x - x^2 - (3 + x - x^2)^2 = 6 + x - x^2 - [(3 + x - x^2)(3 + x - x^2)] =$
$= 6 + x - x^2 - [9 + 3x - 3x^2 + 3x + x^2 - x^3 - 3x^2 - x^3 + x^4] =$
$= 6 + x - x^2 - [9 + 6x - 5x^2 - 2x^3 + x^4] = 6 + x - x^2 - 9 - 6x + 5x^2 + 2x^3 - x^4 =$
$= -3 - 5x + 4x^2 + 2x^3 - x^4 = -x^4 + 2x^3 + 4x^2 - 5x - 3.$

24. $g\big(g(x)\big) = g(x + 1) = (x + 1) + 1 = x + 1 + 1 = x + 2.$

29. $g\big(g(x)\big) = g(1 - x^2) = 1 - (1 - x^2)^2 = 1 - [1 - 2x^2 + x^4] = 1 - 1 + 2x^2 - x^4$
$= 0 + 2x^2 - x^4 = -x^4 + 2x^2.$

30. $g(0) = (0) - 3 = 0 - 3 = -3.$ Hence, $f\big(g(0)\big) = f(-3) = -3.$

32. $g(1) = (1)^2 + 3 = 1 + 3 = 4.$ Hence, $f\big(g(1)\big) = f(4) = (4)^2 = 16.$

34. $f(3) = 3 + (3) - (3)^2 = 3 + 3 - 9 = 6 - 9 = -3.$ Hence, $g\big(f(3)\big) = g(-3) =$
$= (-3) + 1 = -2.$

36. $g(1) = 1 - (1)^2 = 1 - 1 = 0.$ Hence, $g\big(g(1)\big) = g(0) = 1 - (0)^2 = 1 - 0 = 1.$

37. $f\big(g(t)\big) = f(t^2 - 15) = -(t^2 - 15) + 7 = -t^2 + 15 + 7 = -t^2 + 22.$

38. $r(2) = 2^3 = 8.$ Hence, $s\big(r(2)\big) = s(8) = 8 - 7 = 1.$

39. $V\big(A(s)\big) = V(6s^2) = (6s^2)^3 = 6^3 s^6 = 216s^6.$

40. $P(2) = 3 + 2 - 2^2 = 5 - 4 = 1.$ Hence, $D\big(P(2)\big) = D(1) = 1 + 1 = 2.$

41. $f\big(f(\theta)\big) = f(1 + \theta^2) = 1 + (1 + \theta^2)^2 = 1 + 1 + 2\theta^2 + \theta^4 = \theta^4 + 2\theta^2 + 2.$

46. $f\big(g(x)\big) = f\left(\frac{1}{2}x\right) = 2\left(\frac{1}{2}x\right) + 1 = x + 1.$

48. $f\Big(g\big(h(x)\big)\Big) = f\big(g(x^3)\big) = f((x^3)^2) = f(x^6) = x^6 + 1.$

51. $f\Big(g\big(h(x)\big)\Big) = f\big(g(-x)\big) = f(3(-x) + 5) = f(-3x + 5) = (-3x + 5) - 1 = -3x + 4.$