

The Weekly Rigor

No. 197

“A mathematician is a machine for turning coffee into theorems.”

March 31, 2018

39 Problems in Factoring by the Difference of Squares (Part 2)

ANSWERS

1. $(x + 3)(x - 3)$	2. $(x + 4)(x - 4)$	3. $(x + 1)(x - 1)$
4. $(x + 5)(x - 5)$	5. $(x + 6)(x - 6)$	6. $(x + 12)(x - 12)$
7. $(7 + x)(7 - x)$	8. $(1 + x)(1 - x)$	9. $(9 + x)(9 - x)$
10. $(2x + 3)(2x - 3)$	11. $(4x + 1)(4x - 1)$	12. $(7x + 5)(7x - 5)$
13. $(5 + 2x)(5 - 2x)$	14. $(6 + 3x)(6 - 3x)$	15. $(1 + 4x)(1 - 4x)$
16. $(3x + y)(3x - y)$	17. $(8x + y)(8x - y)$	18. $(11x + y)(11x - y)$
19. $(x + 2y)(x - 2y)$	20. $(x + 6y)(x - 6y)$	21. $(x + 11y)(x - 11y)$
22. $(3x + 4y)(3x - 4y)$	23. $(7x + 5y)(7x - 5y)$	24. $(9x + 8y)(9x - 8y)$
25. $(x + y + 2)(x + y - 2)$	26. $(x - y + 9)(x - y - 9)$	27. $(3 + x + y)(3 - x - y)$
28. $(8 + x - y)(8 - x + y)$	29. $4(x + y + 2)(x + y - 2)$	30. $(9x - 9y + 1)(9x - 9y - 1)$
31. $-4(x + 4)$	32. $-3(2x + 1)$	33. $(x^2 + y^2)(x + y)(x - y)$
34. $(3x^2 + 4y^2)(3x^2 - 4y^2)$	35. $(5x^2 + 9y^2)(5x^2 - 9y^2)$	36. $(x + \sqrt{5})(x - \sqrt{5})$
37. $(x + \sqrt{10})(x - \sqrt{10})$	38. $2(x + 5)(x - 5)$	39. $3(3x + 4y)(3x - 4y)$

SELECTED SOLUTIONS

Remark: For these type of factoring problems, keep in mind the “template” pattern of $a^2 - b^2 = (a + b)(a - b)$.

1. $x^2 - 3^2 = (x + 3)(x - 3)$.

3. $x^2 - 1^2 = (x + 1)(x - 1)$.

5. $x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$.

7. $49 - x^2 = 7^2 - x^2 = (7 + x)(7 - x)$.

9. $81 - x^2 = 9^2 - x^2 = (9 + x)(9 - x)$.

11. $16x^2 - 1 = 4^2x^2 - 1^2 = (4x)^2 - 1^2 = (4x + 1)(4x - 1)$.

13. $25 - 4x^2 = 5^2 - 2^2x^2 = 5^2 - (2x)^2 = (5 + 2x)(5 - 2x)$.
15. $1 - 16x^2 = 1^2 - 4^2x^2 = 1^2 - (4x)^2 = (1 + 4x)(1 - 4x)$.
17. $64x^2 - y^2 = 8^2x^2 - y^2 = (8x)^2 - y^2 = (8x + y)(8x - y)$.
19. $x^2 - 4y^2 = x^2 - 2^2y^2 = x^2 - (2y)^2 = (x + 2y)(x - 2y)$.
21. $x^2 - 121y^2 = x^2 - 11^2y^2 = x^2 - (11y)^2 = (x + 11y)(x - 11y)$.
23. $49x^2 - 25y^2 = 7^2x^2 - 5^2y^2 = (7x)^2 - (5y)^2 = (7x + 5y)(7x - 5y)$.
25. $(x + y)^2 - 4 = (x + y)^2 - 2^2 = [(x + y) + 2][(x + y) - 2] = (x + y + 2)(x + y - 2)$.
27. $9 - (x + y)^2 = 3^2 - (x + y)^2 = [3 + (x + y)][3 - (x + y)] = (3 + x + y)(3 - x - y)$.
29. $4(x + y)^2 - 16 = 2^2(x + y)^2 - 4^2 = [2(x + y)]^2 - 4^2 =$
 $= \{2(x + y) + 4\}\{2(x + y) - 4\} = (2x + 2y + 4)(2x + 2y - 4) =$
 $= 2(x + y + 2)2(x + y - 2) = 4(x + y + 2)(x + y - 2)$.
31. $(x + 3)^2 - (x + 5)^2 = [(x + 3) + (x + 5)][(x + 3) - (x + 5)] =$
 $= (x + 3 + x + 5)(x + 3 - x - 5) = (2x + 8)(-2) = -2(2x + 8) = -4(x + 4)$.
33. $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$.
35. $25x^4 - 81y^4 = 5^2(x^2)^2 - 9^2(y^2)^2 = (5x^2)^2 - (9y^2)^2 = (5x^2 + 9y^2)(5x^2 - 9y^2)$.
37. $x^2 - 10 = x^2 - (\sqrt{10})^2 = (x + \sqrt{10})(x - \sqrt{10})$.
39. $27x^2 - 48y^2 = 3(9x^2 - 16y^2) = 3(3^2x^2 - 4^2y^2) = 3[(3x)^2 - (4y)^2] =$
 $= 3(3x + 4y)(3x - 4y)$.

“Only he who never plays, never loses.”