

The Weekly Rigor

No. 198

“A mathematician is a machine for turning coffee into theorems.”

April 7, 2018

30 Problems in Factoring by a Mixture of Methods

PROBLEMS

For each of the following expressions, factor completely by the various methods we have studied.

1. $2x^2 + 15x + 18$

2. $8x^5y^2 + 12xy^3$

3. $x^2 + 1$

4. $n^3 + n$

5. $3x^3 - 2x^2 - 6x + 4$

6. $12x^3 - 8x^2 - 24x + 20$

7. $\theta^2 - 25$

8. $\alpha(\alpha + 7) + 3(\alpha + 7)$

9. $3x^2 + 5x + 4$

10. $121m^2 - n^2$

11. $3x^2 - 75$

12. $4\alpha^2 - 9\beta^2$

13. $5x^3 - 20x + 3x^2 - 12$

14. $12x^3 - 4x^2 - 8x$

15. $12x^6 - 12x^5 + 24x^4 - 24x^3$

16. $5t^2 + 23t + 12$

17. $x^3 + x^2$

18. $\sin^2(x) - \cos^2(x)$
Note: $\sin^2(x) = (\sin(x))^2$

19. $10mn + 50m + 6n^3 + 30n^2$

20. $x^3 - x$

21. $20x^3 + 50x^2 - 30x$

22. $7xy^2 - 28x^5 + y^2 - 4x^4$

23. $\sin^2(\theta) - 1$

24. $24r^4 + 40r^3 + 30r^2 + 50r$

25. $3x^5 - 3x + 5x^4 - 5$

26. $e^{2x} - 1$

27. $3e^{2x} - 2e^x - 5$

28. $e^{3x} - 2e^{2x} + 5e^x - 10$

29. $18e^{3x} - 33e^{2x} + 12e^x$

30. $e^{4x} - 16$

ANSWERS

1. $(2x + 3)(x + 6)$	2. $4xy^2(2x^4 + 3y)$
3. Prime	4. $n(n^2 + 1)$
5. $(3x - 2)(x^2 - 2)$	6. $4(3x^3 - 2x^2 - 6x + 5)$
7. $(\theta + 5)(\theta - 5)$	8. $(\alpha + 7)(\alpha + 3)$
9. Prime	10. $(11m + n)(11m - n)$
11. $3(x + 5)(x - 5)$	12. $(2\alpha + 3\beta)(2\alpha - 3\beta)$
13. $(x + 2)(x - 2)(5x + 3)$	14. $4x(3x + 2)(x - 1)$
15. $12x^3(x - 1)(x^2 + 2)$	16. $(5t + 3)(t + 4)$
17. $x^2(x + 1)$	18. $(\sin(x) + \cos(x))(\sin(x) - \cos(x))$
19. $2(5m + 3n^2)(n + 5)$	20. $x(x + 1)(x - 1)$
21. $10x(2x - 1)(x + 3)$	22. $(7x + 1)(y + 2x^2)(y - 2x^2)$
23. $(\sin(\theta) + 1)(\sin(\theta) - 1)$	24. $2r(4r^2 + 5)(3r + 5)$
25. $(x^2 + 1)(x + 1)(x - 1)(3x + 5)$	26. $(e^x + 1)(e^x - 1)$
27. $(3e^x - 5)(e^x + 1)$	28. $(e^x - 2)(e^{2x} + 5)$
29. $3e^x(2e^x - 1)(3e^x - 4)$	30. $(e^{2x} + 4)(e^x + 2)(e^x - 2)$

SELECTED SOLUTIONS

11. $3x^2 - 75 = 3(x^2 - 25) = 3(x^2 - 5^2) = 3(x + 5)(x - 5).$

19. $10mn + 50m + 6n^3 + 30n^2 = 10m(n + 5) + 6n^2(n + 5) = (10m + 6n^2)(n + 5) = 2(5m + 3n^2)(n + 5).$

23. $\sin^2(\theta) - 1 = (\sin(\theta))^2 - 1 = (\sin(\theta) + 1)(\sin(\theta) - 1).$

29. $18e^{3x} - 33e^{2x} + 12e^x = 3e^x(6e^{2x} - 11e^x + 4) = 3e^x(6e^{2x} - 3e^x - 8e^x + 4) = 3e^x[3e^x(2e^x - 1) - 4(2e^x - 1)] = 3e^x(2e^x - 1)(3e^x - 4).$

“Only he who never plays, never loses.”