

The Weekly Rigor

No. 200

“A mathematician is a machine for turning coffee into theorems.”

April 21, 2018

39 Problems in Solving Equations

(Part 2)

ANSWERS

1. $x = -6$ or $x = \frac{-3}{2}$	2. $x = 0$ or $x = 3$
3. $x = -2$ or $x = \frac{-4}{3}$	4. $n = 0$
5. $x = 0$ or $x = \frac{1}{3}$ or $x = 1$	6. $x = \frac{2}{3}$ or $\sqrt{2}$ or $-\sqrt{2}$
7. $\theta = -5$ or $\theta = 5$	8. $\alpha = -3$ or $\alpha = -7$
9. $a = 3$ or $a = -3$	10. $x = 1$ or $x = 3$
11. $x = 5$ or $x = -5$	12. $x = \frac{-4}{3}$ or $x = -2$
13. $x = -2$ or $x = 2$ or $x = \frac{-3}{5}$	14. $x = 0$ or $x = \frac{-2}{3}$ or $x = 1$
15. $x = 0$ or $x = 1$	16. $t = \frac{-3}{5}$ or $t = -4$
17. $x = 0$ or $x = -1$	18. $x = \frac{-7}{2}$
19. $x = 0$ or $x = -4$ or $x = 4$	20. $t = 0$ or $t = -1$ or $t = 1$
21. $x = 0$ or $x = -3$ or $x = \frac{1}{2}$	22. $x = 2$
23. $x = 4.5$ or $x = -4.5$	24. $x = 0$ or $x = \frac{-5}{3}$
25. $x = -1$ or $x = 1$ or $x = \frac{-5}{3}$	26. $x = 3$ or $x = -3$
27. $x = 0$ or $x = -2$ or $x = 6$	28. $x = -4$ or $x = -5$
29. $x = -2$	30. $x = 0$ or $x = \frac{-2}{3}$ or $x = \frac{2}{3}$
31. $p = \frac{-4}{3}$ or $p = -4$	32. $x = \frac{-8}{7}$ or $x = \frac{8}{7}$
33. $x = \sqrt{5}$ or $x = -\sqrt{5}$	34. $x = \sqrt{\frac{1}{2}}$ or $x = -\sqrt{\frac{1}{2}}$
35. $x = -4$ or $x = \sqrt{\frac{3}{2}}$ or $x = -\sqrt{\frac{3}{2}}$	36. $x = 7$ or $x = 1$
37. $x = 10 - 2\sqrt{5}$ or $x = 10 + 2\sqrt{5}$	38. $x = 2\sqrt{6}$ or $x = -2\sqrt{6}$
39. $x = -2\sqrt{10}$ or $x = 2\sqrt{10}$	

SELECTED SOLUTIONS

$$1. \quad 2x^2 + 15x + 18 = 0 \Rightarrow 2x^2 + 12x + 3x + 18 = 0 \Rightarrow 2x(x + 6) + 3(x + 6) \Rightarrow \\ \Rightarrow (x + 6)(2x + 3) = 0 \Rightarrow x + 6 = 0 \text{ or } 2x + 3 = 0 \Rightarrow x = -6 \text{ or } x = \frac{-3}{2}.$$

$$3. \quad 3x^2 + 6x + 4x + 8 = 0 \Rightarrow 3x(x + 2) + 4(x + 2) = 0 \Rightarrow (x + 2)(3x + 4) = 0 \Rightarrow \\ \Rightarrow x + 2 = 0 \text{ or } 3x + 4 = 0 \Rightarrow x = -2 \text{ or } x = \frac{-4}{3}.$$

$$7. \quad \theta^2 - 25 = 0 \Rightarrow \theta^2 - 5^2 = 0 \Rightarrow (\theta + 5)(\theta - 5) = 0 \Rightarrow \theta + 5 = 0 \text{ or } \theta - 5 = 0 \Rightarrow \\ \Rightarrow \theta = -5 \text{ or } \theta = 5.$$

$$13. \quad 5x^3 - 20x + 3x^2 - 12 = 0 \Rightarrow 5x(x^2 - 4) + 3(x^2 - 4) = 0 \Rightarrow (x^2 - 4)(5x + 3) = 0 \Rightarrow \\ \Rightarrow (x + 2)(x - 2)(5x + 3) = 0 \Rightarrow x + 2 = 0 \text{ or } x - 2 = 0 \text{ or } 5x + 3 = 0 \Rightarrow \\ \Rightarrow x = -2 \text{ or } x = 2 \text{ or } x = \frac{-3}{5}.$$

$$17. \quad x^3 + x^2 = 0 \Rightarrow x^2(x + 1) = 0 \Rightarrow x^2 = 0 \text{ or } x + 1 = 0 \Rightarrow x = 0 \text{ or } x = -1.$$

$$21. \quad 20x^3 + 50x^2 - 30x = 0 \Rightarrow 10x(2x^2 + 5x - 3) = 0 \Rightarrow 10x = 0 \text{ or } 2x^2 + 5x - 3 = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } 2x^2 + 6x - 1x - 3 = 0 \Rightarrow x = 0 \text{ or } 2x(x + 3) - (x + 3) = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } (x + 3)(2x - 1) = 0 \Rightarrow x = 0 \text{ or } x + 3 = 0 \text{ or } 2x - 1 = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } x = -3 \text{ or } x = \frac{1}{2}.$$

$$27. \quad x^5 - 4x^4 - 12x^3 = 0 \Rightarrow x^3(x^2 - 4x - 12) = 0 \Rightarrow x^3 = 0 \text{ or } x^2 - 4x - 12 = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } x^2 + 2x - 6x - 12 = 0 \Rightarrow x = 0 \text{ or } x(x + 2) - 6(x + 2) = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } (x + 2)(x - 6) = 0 \Rightarrow x = 0 \text{ or } x + 2 = 0 \text{ or } x - 6 = 0 \Rightarrow \\ \Rightarrow x = 0 \text{ or } x = -2 \text{ or } x = 6.$$

$$33. \quad x^4 - 25 = 0 \Rightarrow (x^2)^2 - 5^2 = 0 \Rightarrow (x^2 + 5)(x^2 - 5) = 0 \Rightarrow \\ \Rightarrow x^2 + 5 = 0 \text{ or } x^2 - 5 = 0 \Rightarrow x^2 = -5 \text{ or } x^2 = 5 \Rightarrow x = \pm\sqrt{5}.$$

$$37. \quad (x - 10)^2 - 20 = 0 \Rightarrow (x - 10)^2 - (\sqrt{20})^2 = 0 \Rightarrow \\ \Rightarrow [(x - 10) + \sqrt{20}][(x - 10) - \sqrt{20}] = 0 \Rightarrow (x - 10) + \sqrt{20} = 0 \text{ or } (x - 10) - \sqrt{20} = 0 \Rightarrow \\ \Rightarrow x = 10 - \sqrt{20} \text{ or } x = 10 + \sqrt{20} \Rightarrow x = 10 - \sqrt{4 \cdot 5} \text{ or } x = 10 + \sqrt{4 \cdot 5} \Rightarrow \\ \Rightarrow x = 10 - 2\sqrt{5} \text{ or } x = 10 + 2\sqrt{5}.$$

$$39. \quad 3x^2 - 120 = 0 \Rightarrow 3(x^2 - 40) = 0 \Rightarrow x^2 - 40 = 0 \Rightarrow x^2 - (\sqrt{40})^2 = 0 \Rightarrow \\ \Rightarrow (x + \sqrt{40})(x - \sqrt{40}) = 0 \Rightarrow x + \sqrt{40} = 0 \text{ or } x - \sqrt{40} = 0 \Rightarrow \\ \Rightarrow x = -\sqrt{40} \text{ or } x = \sqrt{40} \Rightarrow x = -\sqrt{4 \cdot 10} \text{ or } x = \sqrt{4 \cdot 10} \Rightarrow x = -2\sqrt{10} \text{ or } x = 2\sqrt{10}.$$

“Only he who never plays, never loses.”