The Weekly Rigor

No. 201 "A mathematician is a machine for turning coffee into theorems." April 28, 2018

16 Problems in Reducing Rational Expressions

PROBLEMS

Reduce the following rational expressions to least factors.

1. $\frac{x^2-9}{x+3}$

2. $\frac{x^2-4}{x-2}$

3. $\frac{x^2-16}{x-4}$

4. $\frac{x-5}{x^2-25}$

5. $\frac{x^2+x-12}{x-3}$

6. $\frac{x^2+3x+2}{x^2+4x+3}$

7. $\frac{x^3+5x^2+6x}{x^2+x-2}$

8. $\frac{x^2+5x+6}{x+2}$

9. $\frac{(x+3)^2-9}{x}$

10. $\frac{(x-4)^2-9}{x-7}$

11. $\frac{(x-10)^2-49}{x-17}$

12. $\frac{2x^2+15x+18}{x+6}$

13. $\frac{2x^2+6x}{x+3}$

14. $\frac{3x^2+6x+4x+8}{x+2}$

15. $\frac{6x^3-6x^2-2x^2+2x}{x}$

16. $\frac{9x^4-144x^2}{x}$

ANSWERS

1. $x-3$	2. $x+2$
3. $x+4$	4. $\frac{1}{x+5}$
5. $x+4$	6. $\frac{x+2}{x+3}$
7. $\frac{x(x+3)}{x-1}$	8. $x+3$
9. $x+6$	10. $x-1$
11. $x-3$	12. $2x+3$
13. $2x$	14. $3x+4$
15. $2(x-1)(3x-1)$	16. $9x(x+4)(x-4)$

SELECTED SOLUTIONS

1. $\frac{x^2-9}{x+3}=\frac{x^2-3^2}{x+3}=\frac{(x+3)(x-3)}{x+3}=x-3.$

3. $\frac{x^2-16}{x-4}=\frac{x^2-4^2}{x-4}=\frac{(x+4)(x-4)}{x-4}=x+4.$

5. $\frac{x^2+x-12}{x-3}=\frac{x^2+4x-3x-12}{x-3}=\frac{x(x+4)-3(x+4)}{x-3}=\frac{(x+4)(x-3)}{x-3}=x+4.$

7. $\frac{x^3+5x^2+6x}{x^2+x-2}=\frac{x(x^2+5x+6)}{x^2+x-2}=\frac{x(x^2+3x+2x+6)}{x^2+2x-1x-2}=\frac{x[x(x+3)+2(x+3)]}{x(x+2)-(x+2)}=\frac{x(x+3)(x+2)}{(x+2)(x-1)}=\frac{x(x+3)}{x-1}.$

9. $\frac{(x+3)^2-9}{x}=\frac{(x+3)^2-3^2}{x}=\frac{[(x+3)+3][(x+3)-3]}{x}=\frac{(x+6)x}{x}=x+6.$

11. $\frac{(x-10)^2-49}{x-17}=\frac{(x-10)^2-7^2}{x-17}=\frac{[(x-10)+7][(x-10)-7]}{x-17}=\frac{(x-3)(x-17)}{x-17}=x-3.$

13. $\frac{2x^2+6x}{x+3}=\frac{2x(x+3)}{x+3}=2x.$

15. $\frac{6x^3-6x^2-2x^2+2x}{x}=\frac{6x^3-8x^2+2x}{x}=\frac{2x(3x^2-4x+1)}{x}=2(3x^2-4x+1)=2(3x^2-3x-1x+1)=$
$=2[3x(x-1)-(x-1)]=2(x-1)(3x-1).$