

The Weekly Rigor

No. 202

“A mathematician is a machine for turning coffee into theorems.”

May 5, 2018

16 Problems in Adding Rational Expressions

PROBLEMS

Add the following rational expressions.

1. $\frac{3}{4} + \frac{5}{7}$

2. $\frac{20}{60} + \frac{2}{3}$

3. $\frac{5}{x} + \frac{3}{x}$

4. $\frac{x}{x^2-1} + \frac{1}{x^2-1}$

5. $\frac{x+4}{2x+10} + \frac{5}{x^2-25}$

6. $\frac{7}{x+3} + \frac{11}{x^2-9}$

7. $\frac{7}{x^2-64} + \frac{3}{x+8}$

8. $\frac{x+4}{2x} - \frac{x-1}{x^2}$

9. $\frac{2x-5}{x^2} - \frac{3x+4}{x^3}$

10. $\frac{3}{x^2-9} - \frac{2}{x^2-2x-3}$

11. $\frac{5}{x^2-3x+2} - \frac{1}{x-2}$

12. $\frac{x^2+3x-2}{x^2+3x-10} + \frac{4x+12}{x^2+3x-10}$

13. $\frac{x^2-2x+3}{x^2+7x+12} - \frac{x^2-4x-5}{x^2+7x+12}$

14. $\frac{7}{3x^2-6x} + \frac{x^2}{x^2-4x+4}$

15. $\frac{4x}{2x-1} - \frac{5}{x-6}$

16. $\frac{3}{x+2} + 2$

ANSWERS

1. $\frac{41}{28}$	2. 1
3. $\frac{8}{x}$	4. $\frac{1}{x-1}$
5. $\frac{x^2-x-10}{2(x+5)(x-5)}$	6. $\frac{7x-10}{(x+3)(x-3)}$
7. $\frac{3x-17}{(x+8)(x-8)}$	8. $\frac{x^2+2x+2}{2x^2}$
9. $\frac{2x^2-8x-4}{x^3}$	10. $\frac{1}{(x+3)(x+1)}$
11. $\frac{6-x}{(x-2)(x-1)}$	12. $\frac{x+2}{x-2}$
13. $\frac{2}{x+3}$	14. $\frac{7(x-2)+3x^3}{3x(x-2)^2}$
15. $\frac{4x^2-34x+5}{(x-6)(2x-1)}$	16. $\frac{2x+7}{x+2}$

SELECTED SOLUTIONS

$$1. \frac{3}{4} + \frac{5}{7} = \frac{7}{7} \cdot \frac{3}{4} + \frac{5}{7} \cdot \frac{4}{4} = \frac{21}{28} + \frac{20}{28} = \frac{21+20}{28} = \frac{41}{28}.$$

$$3. \frac{5}{x} + \frac{3}{x} = \frac{5+3}{x} = \frac{8}{x}.$$

$$\begin{aligned} 5. \frac{x+4}{2x+10} + \frac{5}{x^2-25} &= \frac{x+4}{2(x+5)} + \frac{5}{x^2-5^2} = \frac{x+4}{2(x+5)} + \frac{5}{(x+5)(x-5)} = \frac{(x-5)}{(x-5)} \cdot \frac{x+4}{2(x+5)} + \frac{5}{(x+5)(x-5)} \cdot \frac{2}{2} \\ &= \frac{(x-5)(x+4)}{2(x+5)(x-5)} + \frac{10}{2(x+5)(x-5)} = \frac{x^2-x-20+10}{2(x+5)(x-5)} = \frac{x^2-x-10}{2(x+5)(x-5)}. \end{aligned}$$

$$\begin{aligned} 7. \frac{7}{x^2-64} + \frac{3}{x+8} &= \frac{7}{x^2-8^2} + \frac{3}{x+8} = \frac{7}{(x+8)(x-8)} + \frac{3}{x+8} = \frac{7}{(x+8)(x-8)} + \frac{3}{x+8} \cdot \frac{(x-8)}{(x-8)} = \frac{7+3(x-8)}{(x+8)(x-8)} = \frac{7+3x-24}{(x+8)(x-8)} \\ &= \frac{3x-17}{(x+8)(x-8)}. \end{aligned}$$

$$9. \frac{2x-5}{x^2} - \frac{3x+4}{x^3} = \frac{x}{x} \cdot \frac{2x-5}{x^2} - \frac{3x+4}{x^3} = \frac{x(2x-5)-(3x+4)}{x^3} = \frac{2x^2-5x-3x-4}{x^3} = \frac{2x^2-8x-4}{x^3}.$$

$$\begin{aligned} 11. \frac{5}{x^2-3x+2} - \frac{1}{x-2} &= \frac{5}{x^2-2x-1x+2} - \frac{1}{x-2} = \frac{5}{x(x-2)-(x-2)} - \frac{1}{x-2} = \frac{5}{(x-2)(x-1)} - \frac{1}{x-2} = \frac{5}{(x-2)(x-1)} - \frac{1}{x-2} \cdot \frac{(x-1)}{(x-1)} \\ &= \frac{5-(x-1)}{(x-2)(x-1)} = \frac{5-x+1}{(x-2)(x-1)} = \frac{6-x}{(x-2)(x-1)}. \end{aligned}$$

$$13. \frac{x^2-2x+3}{x^2+7x+12} - \frac{x^2-4x-5}{x^2+7x+12} = \frac{x^2-2x+3-(x^2-4x-5)}{x^2+7x+12} = \frac{x^2-2x+3-x^2+4x+5}{x^2+4x+3x+12} = \frac{2x+8}{x(x+4)+3(x+4)} = \frac{2(x+4)}{(x+4)(x+3)} = \frac{2}{x+3}.$$

$$15. \frac{4x}{2x-1} - \frac{5}{x-6} = \frac{x-6}{x-6} \cdot \frac{4x}{2x-1} - \frac{5}{x-6} \cdot \frac{2x-1}{2x-1} = \frac{4x(x-6)-5(2x-1)}{(x-6)(2x-1)} = \frac{4x^2-24x-10x+5}{(x-6)(2x-1)} = \frac{4x^2-34x+5}{(x-6)(2x-1)}$$

“Only he who never plays, never loses.”