

# The Weekly Rigor

No. 204

“A mathematician is a machine for turning coffee into theorems.”

May 19, 2018

## 24 Problems in Multiplying and Dividing Rational Expressions

(Part 2)

### ANSWERS

1. $\frac{15}{28}$	2. $\frac{2}{9}$
3. $\frac{15}{x^2}$	4. $\frac{2x}{9y^5}$
5. $\frac{1}{(x-2)(x-3)}$	6. $\frac{7n^2}{(n+8)(n+6)}$
7. $\frac{p+6}{4}$	8. $\frac{3}{8n^2}$
9. $\frac{x+7}{7}$	10. $\frac{5}{4a(a-1)}$
11. $\frac{3b}{b+8}$	12. $\frac{-(x-5)}{90}$
13. 15	14. $\frac{8(m-1)}{3(m-5)}$
15. $\frac{3}{8}$	16. $\frac{25}{4m-4}$
17. $\frac{84+9x}{1-x}$	18. $\frac{-3-x}{2x^2-10}$
19. $\frac{-1}{3(3+h)}$	20. $\frac{-1}{7(7+h)}$
21. $\frac{-1}{x(x+h)}$	22. $\frac{-1}{5x(x+h)}$
23. $\frac{-3}{(2+x)(2+x+h)}$	24. $\frac{-1}{x(x+\Delta x)}$

### SELECTED SOLUTIONS

$$1. \frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}.$$

$$3. \frac{5}{x} \div \frac{x}{3} = \frac{5}{x} \cdot \frac{3}{x} = \frac{5 \cdot 3}{x^2} = \frac{15}{x^2}.$$

$$\begin{aligned} 5. \frac{x-3}{x^2-4} \cdot \frac{x+2}{x^2-6x+9} &= \frac{x-3}{(x+2)(x-2)} \cdot \frac{x+2}{x^2-3x-3x+9} = \frac{x-3}{(x+2)(x-2)} \cdot \frac{x+2}{x(x-3)-3(x-3)} = \\ &= \frac{x-3}{(x+2)(x-2)} \cdot \frac{x+2}{(x-3)(x+2)} = \frac{(x-3)(x+2)}{(x+2)(x-2)(x-3)(x-3)} = \frac{1}{(x-2)(x-3)}. \end{aligned}$$

$$7. \frac{\frac{2p+12}{4}}{\frac{2p-6}{p-3}} = \frac{\frac{2(p+6)}{2 \cdot 2}}{\frac{2(p-3)}{p-3}} = \frac{\frac{2(p+6)}{2 \cdot 2}}{\frac{2(p-3)}{p-3}} = \frac{\frac{(p+6)}{2}}{\frac{2(p-3)}{p-3}} = \frac{(p+6)}{2} \cdot \frac{(p-3)}{2(p-3)} = \frac{(p+6)(p-3)}{4(p-3)} = \frac{(p+6)\cancel{(p-3)}}{4\cancel{(p-3)}} = \frac{p+6}{4}.$$

$$9. \frac{x+7}{7x+35} \cdot \frac{x^2-3x-40}{x-8} = \frac{x+7}{7(x+5)} \cdot \frac{x^2-8x+5x-40}{x-8} = \frac{x+7}{7(x+5)} \cdot \frac{x(x-8)+5(x-8)}{x-8} = \\ = \frac{x+7}{7(x+5)} \cdot \frac{(x-8)(x+5)}{x-8} = \frac{(x+7)(x-8)(x+5)}{7(x+5)(x-8)} = \frac{(x+7)\cancel{(x-8)}\cancel{(x+5)}}{7\cancel{(x+5)}\cancel{(x-8)}} = \frac{x+7}{7}.$$

$$11. \frac{3b^2+18b}{b+6} \cdot \frac{1}{b+8} = \frac{3b(b+6)}{b+6} \cdot \frac{1}{b+8} = \frac{3b(b+6)}{(b+6)(b+8)} = \frac{3b\cancel{(b+6)}}{\cancel{(b+6)}(b+8)} = \frac{3b}{b+8}.$$

$$13. \frac{45x^2}{x-9} \cdot \frac{x^2-5x-36}{3x^3+12x^2} = \frac{45x^2}{x-9} \cdot \frac{x^2-9x+4x-36}{3x^2(x+4)} = \frac{45x^2}{x-9} \cdot \frac{x(x-9)+4(x-9)}{3x^2(x+4)} = \frac{45x^2}{x-9} \cdot \frac{(x-9)(x+4)}{3x^2(x+4)} = \\ = \frac{45x^2(x-9)(x+4)}{3x^2(x-9)(x+4)} = \frac{3 \cdot 15x^2(x-9)(x+4)}{3x^2(x-9)(x+4)} = \frac{3 \cdot 15\cancel{x^2}\cancel{(x-9)}\cancel{(x+4)}}{3\cancel{x^2}\cancel{(x-9)}\cancel{(x+4)}} = 15.$$

$$15. \frac{\frac{1}{4} + \frac{5}{4}}{\frac{4}{4}} = \frac{\frac{1+5}{4}}{\frac{4}{4}} = \frac{\frac{6}{4}}{\frac{4}{4}} = \frac{\frac{3 \cdot 2}{4}}{\frac{2 \cdot 2}{4}} = \frac{\frac{3 \cdot \cancel{2}}{4}}{\frac{2 \cdot \cancel{2}}{4}} = \frac{\frac{3}{4}}{\frac{2}{4}} = \frac{3}{2} \cdot \frac{1}{4} = \frac{3 \cdot 1}{2 \cdot 4} = \frac{3}{8}.$$

$$17. \frac{\frac{25}{18} + \frac{x+1}{36}}{\frac{1}{2} - \frac{x+1}{36}} = \frac{\frac{25}{2} + \frac{x+1}{18} \cdot \frac{3}{3}}{\frac{2}{2} - \frac{x+1}{36}} = \frac{\frac{25}{2} + \frac{3(x+1)}{36}}{\frac{2-(x+1)}{36}} = \frac{\frac{25+3(x+1)}{36}}{\frac{2-x-1}{36}} = \frac{\frac{25+3x+3}{36}}{\frac{1-x}{36}} = \frac{28+3x}{12} \cdot \frac{36}{1-x} = \\ = \frac{28+3x}{12} \cdot \frac{3 \cdot 12}{1-x} = \frac{28+3x}{12} \cdot \frac{3 \cdot \cancel{12}}{1-x} = \frac{3(28+3x)}{1-x} = \frac{84+9x}{1-x}.$$

$$19. \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \frac{\frac{3}{3} \cdot \frac{1}{3+h} - \frac{1}{3} \cdot \frac{(3+h)}{(3+h)}}{h} = \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \frac{\frac{3-3-h}{3(3+h)}}{h} = \frac{\frac{-h}{3(3+h)}}{\frac{h}{1}} = \frac{-h}{3(3+h)} \cdot \frac{1}{h} = \\ = \frac{-\cancel{h}}{3(3+h)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{3(3+h)}.$$

$$21. \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x}{x} \cdot \frac{1}{x+h} - \frac{1}{x} \cdot \frac{(x+h)}{(x+h)}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\frac{x-x-h}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{\frac{h}{1}} = \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \\ = \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{x(x+h)}.$$

$$23. \frac{\frac{1-(x+h)}{2+(x+h)} - \frac{1-x}{2+x}}{h} = \frac{\frac{1-x-h}{2+x+h} - \frac{1-x}{2+x}}{h} = \frac{\frac{(2+x) \cdot \frac{1-x-h}{2+x+h} - \frac{1-x}{2+x} \cdot \frac{(2+x+h)}{(2+x+h)}}{h}}{h} = \frac{\frac{(2+x)(1-x-h) - (1-x)(2+x+h)}{(2+x)(2+x+h)}}{h} = \\ = \frac{\frac{2-2x-2h+x-x^2-xh-2-x-h+2x+x^2+xh}{(2+x)(2+x+h)}}{h} = \frac{\frac{2-2x-2h+x-x^2-xh-2-x-h+2x+x^2+xh}{(2+x)(2+x+h)}}{h} = \frac{-2h-h}{(2+x)(2+x+h)} = \\ = \frac{-3h}{(2+x)(2+x+h)} = \frac{\frac{-3h}{(2+x)(2+x+h)}}{\frac{h}{1}} = \frac{-3h}{(2+x)(2+x+h)} \cdot \frac{1}{h} = \frac{-3\cancel{h}}{(2+x)(2+x+h)} \cdot \frac{1}{\cancel{h}} = \frac{-3}{(2+x)(2+x+h)}.$$

“Only he who never plays, never loses.”