

The Weekly Rigor

No. 208

“A mathematician is a machine for turning coffee into theorems.”

June 16, 2018

12 Problems in Partial Fractions (Part 2)

SELECTED SOLUTIONS

$$1. \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$(x+1)(x-1) \frac{1}{(x+1)(x-1)} = \left[\frac{A}{x+1} + \frac{B}{x-1} \right] (x+1)(x-1)$$

$$\frac{(x+1)(x-1)}{(x+1)(x-1)} = \frac{A}{x+1} (x+1)(x-1) + \frac{B}{x-1} (x+1)(x-1)$$

$$\frac{(x+1)(x-1)}{(x+1)(x-1)} = \frac{A(x+1)(x-1)}{x+1} + \frac{B(x+1)(x-1)}{x-1}$$

$$1 = A(x-1) + B(x+1)$$

Let $x = 1$:

$$1 = A(1-1) + B(1+1)$$

$$1 = A(0) + B(2)$$

$$1 = 2B$$

$$\frac{1}{2} = \frac{2B}{2}$$

$$\frac{1}{2} = B$$

Let $x = -1$:

$$1 = A(-1-1) + B(-1+1)$$

$$1 = A(-2) + B(0)$$

$$1 = -2A$$

$$\frac{1}{-2} = \frac{-2A}{-2}$$

$$\frac{-1}{2} = A$$

Therefore,

$$\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{-1}{2} \frac{1}{x+1} + \frac{1}{2} \frac{1}{x-1}$$

$$3. \frac{x+7}{x^2-x-6} = \frac{x+7}{x^2-3x+2x-6} = \frac{x+7}{x(x-3)+2(x-3)} = \frac{x+7}{(x-3)(x+2)}$$

$$\frac{x+7}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$(x-3)(x+2) \frac{x+7}{(x-3)(x+2)} = \left[\frac{A}{x-3} + \frac{B}{x+2} \right] (x-3)(x+2)$$

$$\frac{(x-3)(x+2)(x+7)}{(x-3)(x+2)} = \frac{A}{x-3} (x-3)(x+2) + \frac{B}{x+2} (x-3)(x+2)$$

$$\frac{\cancel{(x-3)}\cancel{(x+2)}(x+7)}{\cancel{(x-3)}\cancel{(x+2)}} = \frac{A\cancel{(x-3)}(x+2)}{\cancel{x-3}} + \frac{B(x-3)\cancel{(x+2)}}{\cancel{x+2}}$$

$$x+7 = A(x+2) + B(x-3)$$

Let $x = -2$:

$$-2+7 = A(-2+2) + B(-2-3)$$

$$5 = A(0) + B(-5)$$

$$5 = -5B$$

$$\frac{5}{-5} = \frac{-5B}{-5}$$

$$-1 = B$$

Let $x = 3$:

$$3+7 = A(3+2) + B(3-3)$$

$$10 = A(5) + B(0)$$

$$10 = 5A$$

$$\frac{2 \cdot 5}{5} = \frac{5A}{5}$$

$$2 = A$$

Therefore,

$$\frac{x+7}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{2}{x-3} + \frac{-1}{x+2}$$

“Only he who never plays, never loses.”