12 Problems in Partial Fractions
(Part 2)

SELECTED SOLUTIONS

1. $\dfrac{1}{x^2-1} = \dfrac{1}{(x+1)(x-1)}$

$\dfrac{1}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$

$(x+1)(x-1)\dfrac{1}{(x+1)(x-1)} = \left[\dfrac{A}{x+1} + \dfrac{B}{x-1}\right](x+1)(x-1)$

$\dfrac{(x+1)(x-1)}{(x+1)(x-1)} = \dfrac{A}{x+1}(x+1)(x-1) + \dfrac{B}{x-1}(x+1)(x-1)$

$\dfrac{\cancel{(x+1)(x-1)}}{\cancel{(x+1)(x-1)}} = \dfrac{A\cancel{(x+1)}(x-1)}{\cancel{x+1}} + \dfrac{B(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$1 = A(x-1) + B(x+1)$

Let $x = 1$:

$1 = A(1-1) + B(1+1)$

$1 = A(0) + B(2)$

$1 = 2B$

$\dfrac{1}{2} = \dfrac{\cancel{2}B}{\cancel{2}}$

$\dfrac{1}{2} = B$

Let $x = -1$:

$1 = A(-1-1) + B(-1+1)$

$1 = A(-2) + B(0)$

$1 = -2A$

$\dfrac{1}{-2} = \dfrac{\cancel{-2}A}{\cancel{-2}}$

$\dfrac{-1}{2} = A$

Therefore,

$$\dfrac{1}{x^2-1} = \dfrac{A}{x+1} + \dfrac{B}{x-1} = \dfrac{\frac{-1}{2}}{x+1} + \dfrac{\frac{1}{2}}{x-1}$$

3. $\dfrac{x+7}{x^2-x-6} = \dfrac{x+7}{x^2-3x+2x-6} = \dfrac{x+7}{x(x-3)+2(x-3)} = \dfrac{x+7}{(x-3)(x+2)}$

$\dfrac{x+7}{(x-3)(x+2)} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$

$(x-3)(x+2)\dfrac{x+7}{(x-3)(x+2)} = \left[\dfrac{A}{x-3} + \dfrac{B}{x+2}\right](x-3)(x+2)$

$\dfrac{(x-3)(x+2)(x+7)}{(x-3)(x+2)} = \dfrac{A}{x-3}(x-3)(x+2) + \dfrac{B}{x+2}(x-3)(x+2)$

$\dfrac{\cancel{(x-3)(x+2)}(x+7)}{\cancel{(x-3)(x+2)}} = \dfrac{A\cancel{(x-3)}(x+2)}{\cancel{x-3}} + \dfrac{B(x-3)\cancel{(x+2)}}{\cancel{x+2}}$

$x + 7 = A(x+2) + B(x-3)$

Let $x = -2$:

$-2 + 7 = A(-2+2) + B(-2-3)$

$5 = A(0) + B(-5)$

$5 = -5B$

$\dfrac{5}{-5} = \dfrac{-5B}{-5}$

$-1 = B$

Let $x = 3$:

$3 + 7 = A(3+2) + B(3-3)$

$10 = A(5) + B(0)$

$10 = 5A$

$\dfrac{2\cdot\cancel{5}}{\cancel{5}} = \dfrac{\cancel{5}A}{\cancel{5}}$

$2 = A$

Therefore,

$$\dfrac{x+7}{x^2-x-6} = \dfrac{A}{x-3} + \dfrac{B}{x+2} = \dfrac{2}{x-3} + \dfrac{-1}{x+2}$$