

The Weekly Rigor

No. 209

“A mathematician is a machine for turning coffee into theorems.”

June 23, 2018

12 Problems in Partial Fractions

(Part 3)

$$5. \frac{1}{4x^2-9} = \frac{1}{(2x+3)(2x-3)}$$

$$\frac{1}{(2x+3)(2x-3)} = \frac{A}{2x+3} + \frac{B}{2x-3}$$

$$(2x+3)(2x-3) \frac{1}{(2x+3)(2x-3)} = \left[\frac{A}{2x+3} + \frac{B}{2x-3} \right] (2x+3)(2x-3)$$

$$\frac{(2x+3)(2x-3)}{(2x+3)(2x-3)} = \frac{A}{2x+3} (2x+3)(2x-3) + \frac{B}{2x-3} (2x+3)(2x-3)$$

$$\frac{(2x+3)(2x-3)}{(2x+3)(2x-3)} = \frac{A(2x+3)(2x-3)}{2x+3} + \frac{B(2x+3)(2x-3)}{2x-3}$$

$$1 = A(2x-3) + B(2x+3)$$

$$\text{Let } x = \frac{3}{2}:$$

$$1 = A\left(2 \cdot \frac{3}{2} - 3\right) + B\left(2 \cdot \frac{3}{2} + 3\right)$$

$$1 = A(0) + B(6)$$

$$1 = 6B$$

$$\frac{1}{6} = B$$

$$\text{Let } x = -\frac{3}{2}:$$

$$1 = A\left(2 \cdot \frac{-3}{2} - 3\right) + B\left(2 \cdot \frac{-3}{2} + 3\right)$$

$$1 = A(-6) + B(0)$$

$$1 = -6A$$

$$\frac{-1}{6} = A$$

Therefore,

$$\frac{1}{4x^2-9} = \frac{A}{2x+3} + \frac{B}{2x-3} = \frac{-\frac{1}{6}}{2x+3} + \frac{\frac{1}{6}}{2x-3}$$

$$7. \frac{1}{2x^2+x} = \frac{1}{x(2x+1)}$$

$$\frac{1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

$$x(2x+1) \frac{1}{x(2x+1)} = \left[\frac{A}{x} + \frac{B}{2x+1} \right] x(2x+1)$$

$$\frac{x(2x+1)}{x(2x+1)} = \frac{A}{x} x(2x+1) + \frac{B}{2x+1} x(2x+1)$$

$$\frac{x(2x+1)}{x(2x+1)} = \frac{Ax(2x+1)}{x} + \frac{Bx(2x+1)}{2x+1}$$

$$1 = A(2x+1) + Bx$$

$$\text{Let } x = -\frac{1}{2}:$$

$$\text{Let } x = 0:$$

$$1 = A\left(2 \cdot \frac{-1}{2} + 1\right) + B\left(\frac{-1}{2}\right)$$

$$1 = A(2 \cdot 0 + 1) + B(0)$$

$$1 = A(0) + \frac{-1}{2}B$$

$$1 = A(1) + B(0)$$

$$1 = \frac{-1}{2}B$$

$$1 = A$$

$$-2 = B$$

Therefore,

$$\frac{1}{2x^2+x} = \frac{A}{x} + \frac{B}{2x+1} = \frac{1}{x} + \frac{-2}{2x+1}$$

“Only he who never plays, never loses.”