

# The Weekly Rigor

No. 210

“A mathematician is a machine for turning coffee into theorems.”

June 30, 2018

## 12 Problems in Partial Fractions

(Part 4)

$$9. \frac{3}{x^2+x-2} = \frac{3}{x^2-x+2x-2} = \frac{3}{x(x-1)+2(x-1)} = \frac{3}{(x-1)(x+2)}$$

$$\frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$(x-1)(x+2) \frac{3}{(x-1)(x+2)} = \left[ \frac{A}{x-1} + \frac{B}{x+2} \right] (x-1)(x+2)$$

$$\frac{3(x-1)(x+2)}{(x-1)(x+2)} = \frac{A}{x-1} (x-1)(x+2) + \frac{B}{x+2} (x-1)(x+2)$$

$$\frac{3(x-1)(x+2)}{\cancel{(x-1)(x+2)}} = \frac{A(x-1)(x+2)}{x-1} + \frac{B(x-1)(x+2)}{x+2}$$

$$3 = A(x+2) + B(x-1)$$

Let  $x = -2$ :

$$3 = A(-2+2) + B(-2-1)$$

$$3 = A(0) + B(-3)$$

$$3 = -3B$$

$$-1 = B$$

Let  $x = 1$ :

$$3 = A(1+2) + B(1-1)$$

$$3 = A(3) + B(0)$$

$$3 = 3A$$

$$1 = A$$

Therefore,

$$\frac{3}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{1}{x-1} + \frac{-1}{x+2}$$

$$11. \frac{x+2}{x^2-4x} = \frac{x+2}{x(x-4)}$$

$$\frac{x+2}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$$

$$x(x-4) \frac{x+2}{x(x-4)} = \left[ \frac{A}{x} + \frac{B}{x-4} \right] x(x-4)$$

$$\frac{x(x-4)(x+2)}{x(x-4)} = \frac{A}{x} x(x-4) + \frac{B}{x-4} x(x-4)$$

$$\frac{x(x-4)(x+2)}{x(x-4)} = \frac{A(x-4)}{x} + \frac{Bx(x-4)}{x-4}$$

$$x + 2 = A(x - 4) + Bx$$

Let  $x = 4$ :

$$4 + 2 = A(4 - 4) + B(4)$$

$$6 = A(0) + 4B$$

$$6 = 4B$$

$$\frac{3}{2} = B$$

Let  $x = 0$ :

$$0 + 2 = A(0 - 4) + B(0)$$

$$2 = A(-4) + B(0)$$

$$2 = -4A$$

$$\frac{-1}{2} = A$$

Therefore,

$$\frac{x+2}{x^2-4x} = \frac{A}{x} + \frac{B}{x-4} = \frac{-1}{2x} + \frac{3}{2(x-4)}$$

“Only he who never plays, never loses.”