

The Weekly Rigor

No. 212

“A mathematician is a machine for turning coffee into theorems.”

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16 Problems in Finding Linear Functions

(Part 2)

SELECTED SOLUTIONS

1. Using the slope-intercept form equation, $y = mx + b$, where m specifies the slope of the line and b the y -intercept, we have by substitution $y = 2x + 7$.

3. Step 1: Using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, we can let $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (4, 7)$. Hence, $m = \frac{7 - 5}{4 - 2} = \frac{2}{2} = 1$. (We could also have chosen $(x_1, y_1) = (4, 7)$ and $(x_2, y_2) = (2, 5)$. By the slope formula we get the same slope: $m = \frac{5 - 7}{2 - 4} = \frac{-2}{-2} = 1$.)

Step 2: Next we can use the slope-intercept form equation, $y = mx + b$, to find what b is. We can use either given point and substitute in the values of x , y , and m :

$$y = mx + b$$

$$5 = 1(2) + b$$

$$3 = b$$

Now that we know the slope m and the y -intercept b , we can write the equation of the line:

$$y = x + 3$$

Note that if we had used the other point, $(4, 7)$, we would get the same answer:

$$y = mx + b$$

$$7 = 1(4) + b$$

$$3 = b$$

Hence, the equation is again

$$y = x + 3$$

5. The slope of the given line is -2 , and our line is parallel to the given line. Hence, our slope is the same, namely, $m = -2$. Now we can use the slope-intercept form equation, $y = mx + b$, to find what b is. We can use the given point, $(-1, -1)$, and substitute in the values of x , y , and m :

$$y = mx + b$$

$$-1 = -2(-1) + b$$

$$-1 = 2 + b$$

$$-3 = b$$

Now that we know the slope m and the y -intercept b , we can write the equation of the line:

$$y = -2x - 3$$

7. The y -value of any x -intercept is 0 . Hence, we were actually given the point $(\frac{-1}{5}, 0)$. On the other hand, the x -value of any y -intercept is 0 . So, we were actually given the point $(0, 2)$. Thus, now we can proceed as in problem 3.

Step 1: Using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, we can let $(x_1, y_1) = (\frac{-1}{5}, 0)$ and $(x_2, y_2) = (0, 2)$. Hence, $m = \frac{2 - 0}{0 - \frac{-1}{5}} = \frac{2 - 0}{0 + \frac{1}{5}} = \frac{2}{(\frac{1}{5})} = \frac{(\frac{2}{1})}{(\frac{1}{5})} = \frac{2}{1} \cdot \frac{5}{1} = 10$.

Step 2: Next we can use the slope-intercept form equation, $y = mx + b$, to find what b is. We can use either given point and substitute in the values of x , y , and m :

$$y = mx + b$$

$$0 = 10\left(\frac{-1}{5}\right) + b$$

$$0 = -2 + b$$

$$2 = b$$

Now that we know the slope m and the y -intercept b , we can write the equation of the line:

$$y = 10x + 2$$

(Note that we were actually given in the original problem what b , the y -intercept, is equal to.)

“Only he who never plays, never loses.”