

The Weekly Rigor

No. 216

“A mathematician is a machine for turning coffee into theorems.”

August 11, 2018

16 Problems in Completing the Square (Part 1)

PROBLEMS

Rewrite the following expressions by completing the square.

1. $x^2 + 2x + 5$

2. $x^2 - 12x + 19$

3. $x^2 + 4x - 3$

4. $x^2 + 16x + 9$

5. $x^2 - 6x + 13$

6. $x^2 + 3x + 5$

7. $x^2 + 5x - 3$

8. $x^2 + 3x + 2$

9. $3x^2 + 6x + 4$

10. $2x^2 + 4x - 1$

11. $2x^2 - 12x + 3$

12. $4x^2 + 16x + 25$

13. $4x^2 - 8x + 9$

14. $3x^2 + 12x + 4$

15. $4x^2 + 24x + 41$

16. $5x^2 - 10x + 11$

ANSWERS

1. $(x + 1)^2 + 4$	2. $(x - 6)^2 - 17$
3. $(x + 2)^2 - 7$	4. $(x + 8)^2 - 55$
5. $(x - 3)^2 + 4$	6. $\left(x + \frac{3}{2}\right)^2 + \frac{11}{4}$
7. $\left(x + \frac{5}{2}\right)^2 - \frac{37}{4}$	8. $\left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$
9. $3(x + 1)^2 + 1$	10. $2(x + 1)^2 - 3$
11. $2(x - 3)^2 - 15$	12. $4(x + 2)^2 + 9$
13. $4(x - 1)^2 + 5$	14. $3(x + 2)^2 - 8$
15. $4(x + 3)^2 + 5$	16. $5(x - 1)^2 + 6$

SELECTED SOLUTIONS

3. $x^2 + 4x - 3 = (x^2 + 4x + \underline{\hspace{2cm}}) - 3 - \underline{\hspace{2cm}}$

$4 \cdot \frac{1}{2} = \frac{4}{2} = 2$. $(2)^2 = 4$. So, we use 4:

$$(x^2 + 4x + 4) - 3 - 4$$

$$(x + 2)^2 - 7$$

Check: $(x + 2)^2 - 7 = x^2 + 4x + 4 - 7 = x^2 + 4x - 3$. ✓

5. $x^2 - 6x + 13 = (x^2 - 6x + \underline{\hspace{2cm}}) + 13 - \underline{\hspace{2cm}}$

$-6 \cdot \frac{1}{2} = \frac{-6}{2} = -3$. $(-3)^2 = 9$. So, we use 9:

$$(x^2 - 6x + 9) + 13 - 9$$

$$(x - 3)^2 + 4$$

Check: $(x - 3)^2 + 4 = x^2 - 6x + 9 + 4 = x^2 - 6x + 13$. ✓

“Only he who never plays, never loses.”