

The Weekly Rigor

No. 217

“A mathematician is a machine for turning coffee into theorems.”

August 18, 2018

16 Problems in Completing the Square

(Part 2)

$$7. x^2 + 5x - 3 = (x^2 + 5x + \underline{\quad}) - 3 - \underline{\quad}$$

$$5 \cdot \frac{1}{2} = \frac{5}{2}. \left(\frac{5}{2}\right)^2 = \frac{25}{4}. \text{ So, we use } \frac{25}{4}:$$

$$\left(x^2 + 5x + \frac{25}{4}\right) - 3 - \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{12}{4} - \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{37}{4}$$

$$\text{Check: } \left(x + \frac{5}{2}\right)^2 - \frac{37}{4} = x^2 + 5x + \frac{25}{4} - \frac{37}{4} = x^2 + 5x - \frac{12}{4} = x^2 + 5x - 3. \checkmark$$

$$9. 3x^2 + 6x + 4 = 3(x^2 + 2x) + 4 = 3(x^2 + 2x + \underline{\quad}) + 4 - 3(\underline{\quad})$$

$$2 \cdot \frac{1}{2} = \frac{2}{2} = 1. (1)^2 = \underline{1}. \text{ So, we use } 1:$$

$$3(x^2 + 2x + \underline{1}) + 4 - 3(\underline{1})$$

$$3(x + 1)^2 + 4 - 3$$

$$3(x + 1)^2 + 1$$

$$\text{Check: } 3(x + 1)^2 + 1 = 3(x^2 + 2x + 1) + 1 = 3x^2 + 6x + 3 + 1 = 3x^2 + 6x + 4. \checkmark$$

$$11. 2x^2 - 12x + 3 = 2(x^2 - 6x) + 3 = 2(x^2 - 6x + \underline{\quad}) + 3 - 2(\underline{\quad})$$

$$-6 \cdot \frac{1}{2} = \frac{-6}{2} = -3. (-3)^2 = \underline{9}. \text{ So, we use 9:}$$

$$2(x^2 - 6x + \underline{9}) + 3 - 2(\underline{9})$$

$$2(x - 3)^2 + 3 - 18$$

$$2(x - 3)^2 - 15$$

Check: $2(x - 3)^2 - 15 = 2(x^2 - 6x + 9) - 15 = 2x^2 - 12x + 18 - 15 = 2x^2 - 12x + 3. \checkmark$

$$13. 4x^2 - 8x + 9 = 4(x^2 - 2x) + 9 = 4(x^2 - 2x + \underline{\quad}) + 9 - 4(\underline{\quad})$$

$$-2 \cdot \frac{1}{2} = \frac{-2}{2} = -1. (-1)^2 = \underline{1}. \text{ So, we use 1:}$$

$$4(x^2 - 2x + \underline{1}) + 9 - 4(\underline{1})$$

$$4(x - 1)^2 + 9 - 4$$

$$4(x - 1)^2 + 5$$

Check: $4(x - 1)^2 + 5 = 4(x^2 - 2x + 1) + 5 = 4x^2 - 8x + 4 + 5 = 4x^2 - 8x + 9. \checkmark$

$$15. 4x^2 + 24x + 41 = 4(x^2 + 6x) + 41 = 4(x^2 + 6x + \underline{\quad}) + 41 - 4(\underline{\quad})$$

$$6 \cdot \frac{1}{2} = \frac{6}{2} = 3. 3^2 = \underline{9}. \text{ So, we use 9:}$$

$$4(x^2 + 6x + \underline{9}) + 41 - 4(\underline{9})$$

$$4(x + 3)^2 + 41 - 36$$

$$4(x + 3)^2 + 5$$

Check: $4(x + 3)^2 + 5 = 4(x^2 + 6x + 9) + 5 = 4x^2 + 24x + 36 + 5 = 4x^2 + 24x + 41. \checkmark$

“Only he who never plays, never loses.”