The Weekly Rigor

No. 219 "A mathematician is a machine for turning coffee into theorems." September 1, 2018

20 Problems in Function Notation and Classification

(Part 2)

ANSWERS

1. $-7; 11; -11$	2. $-9; 47; 3$
3. 2	4. 8
5. $(-1,3)$	6. $(2,8)$
7. Odd	8. Neither
9. Neither	10. Even
11. Odd	12. Neither
13. Even	14. Neither
15. Even	16. Even
17. Odd	18. Odd
19. Neither	20. Even

SELECTED SOLUTIONS

1. a. $f(-2) = 2(-2) - 3 = -4 - 3 = -7$

   b. $f(7) = 2(7) - 3 = 14 - 3 = 11$

   c. $f(-4) = 2(-4) - 3 = -8 - 3 = -11$

3. Since $j(x) = 12$, we have by substitution, $12 = 5x + 2$. Solving for *x* we have:

$$10 = 5x$$

$$2 = x$$

5. "$f(-1) = 3$" means that the input value is -1 and the output value 3. Therefore, in coordinate points we write $(-1,3)$.

7. $f(-x) = (-x)^3 = (-1 \cdot x)^3 = (-1)^3 x^3 = -1x^3 = -x^3 = -f(x)$. Therefore, this function is odd.

9. $f(-x) = (-x)^2 + (-x) = (-1 \cdot x)^2 + (-1 \cdot x) = (-1)^2 x^2 - x = 1 \cdot x^2 - x =$
$= x^2 - x$. But $x^2 - x \neq x^2 + x = f(x)$. Hence, this function is not even. On the other hand, $x^2 - x \neq -(x^2 + x) = -f(x)$. So, this function is not odd. Therefore, this function is neither even nor odd.

11. $f(-x) = (-x)^3 + 4(-x) = (-1 \cdot x)^3 + 4(-1 \cdot x) = (-1)^3 x^3 + 4(-1)x =$
$= -1x^3 - 4x = -(x^3 + 4x) = -f(x)$. Therefore, this function is odd.

13. $f(-x) = \sqrt{(-x)^4 - (-x)^2} + 4 = \sqrt{(-1 \cdot x)^4 - (-1 \cdot x)^2} + 4 =$
$= \sqrt{(-1)^4 (x)^4 - (-1)^2 (-x)^2} + 4 = \sqrt{1 \cdot (x)^4 - 1 \cdot (-x)^2} + 4 = \sqrt{x^4 - x^2} + 4 = f(x)$.
Therefore, this function is even.

15. $f(-x) = |-x| + 4 = |-1 \cdot x| + 4 = |-1| \cdot |x| + 4 = 1 \cdot |x| + 4 = |x| + 4 = f(x)$.
Therefore, this function is even.

17. $f(-x) = \sqrt[3]{-x} = \sqrt[3]{-1 \cdot x} = \sqrt[3]{-1} \cdot \sqrt[3]{x} = -1\sqrt[3]{x} = -\sqrt[3]{x} = -f(x)$. Therefore, this function is odd.

19. $f(-x) = (-x)^3 - (-x)^2 + 4(-x) + 2 = (-1 \cdot x)^3 - (-1 \cdot x)^2 + 4(-1 \cdot x) + 2 =$
$= (-1)^3 x^3 - (-1)^2 x^2 + 4(-1)x + 2 = -1 \cdot x^3 - 1 \cdot x^2 - 4x + 2 = -x^3 - x^2 - 4x + 2$.
But $-x^3 - x^2 - 4x + 2 \neq x^3 - x^2 + 4x + 2 = f(x)$. Hence, this function is not even. On the other hand, $-x^3 - x^2 - 4x + 2 \neq -(x^3 - x^2 + 4x + 2) = -f(x)$. So, this function is not odd. Therefore, this function is neither even nor odd.