The Weekly Rigor

No. 236 "A mathematician is a machine for turning coffee into theorems." December 29, 2018

26 Problems in Composite Functions and Interval Notation

(Part 2)

For problems 21-26, write the inequalities in interval notation.

21. $3 \leq x \leq 15$

22. $-8 \leq x < 25$

23. $x \geq 0$

24. $y \geq 0$

25. $0 \leq \theta < 2\pi$

26. All real numbers ("$\mathbb{R}$").

ANSWERS

1. $-9x^4+3; (-9x+3)^4$	2. $2(x+2)-5; (2x-5)+2$
3. $(x-3)^2+7; (x^2+7)-3$	4. $4x^2+3; (4x+3)^2$ or $16x^2+24x+9$
5. $(x-1)^2+2(x-1)-8$ or x^2-9	6. $(x+h)+1$ or $x+h+1$
7. $(x+h)^2$ or $x^2+2xh+h^2$	8. $\frac{1}{x+h}$
9. $\sqrt{x+h}$	10. $\frac{1}{\sqrt{x+h}}$
11. $22; 16$	12. $-18; 72$
13. $-6; 26$	14. $-8; 100$
15. $(x+h)^2$	16. $2x-5$
17. $x-5$	18. $5x+1$
19. $(-3x-2)^3$	20. $x+h$
21. $[3,15]$	22. $[-8,25)$
23. $[0,\infty)$	24. $[0,\infty)$
25. $[0,2\pi)$	26. $(-\infty,\infty)$

SELECTED SOLUTIONS

1. $(f \circ g)(x) = f\big(g(x)\big) = f(x^4) = -9(x^4)+3 = -9x^4+3$. $(g \circ f)(x) = g\big(f(x)\big) =$
$= g(-9x+3) = (-9x+3)^4$.

11. $(f \circ g)(3) = f\big(g(3)\big)$. $g(3) = 3^2 = 9$. Hence, $f\big(g(3)\big) = f(9) = 3(9)-5 = 22$.
$(g \circ f)(3) = g\big(f(3)\big)$. $f(3) = 3(3)-5 = 4$. Hence, $g\big(f(3)\big) = g(4) = 4^2 = 16$.

12. $(f \circ g)(10) = f\big(g(10)\big)$. $g(10) = \sqrt{10-9} = 1$. Hence, $f\big(g(10)\big) = f(1) =$
$= -9(1)-9 = -18$. $(f \circ f)(0) = f\big(f(0)\big)$. $f(0) = -9(0)-9 = -9$. Hence,
$f\big(f(0)\big) = f(-9) = -9(-9)-9 = 81-9 = 72$.

15. $(f \circ g)(x) = f\big(g(x)\big)$. $g(x) = x+h$. Hence, $f\big(g(x)\big) = f(x+h) = (x+h)^2$.

16. $(f \circ g)(x) = f\big(g(x)\big) = (2x-5)^2$. But $f(x) = x^2$. Hence, $f\big(g(x)\big) = \big(g(x)\big)^2$.
Consequently, $\big(g(x)\big)^2 = (2x-5)^2$. Therefore, $g(x) = 2x-5$.

20. $(f \circ g)(x) = f\big(g(x)\big) = (x+h)^2 + (x+h)$. But $f(x) = x^2+x$. Hence,
$f\big(g(x)\big) = \big(g(x)\big)^2 + \big(g(x)\big)$. Consequently, $\big(g(x)\big)^2 + \big(g(x)\big) = (x+h)^2 + (x+h)$.
Therefore, $g(x) = x+h$.

"Only he who never plays, never loses."