The Weekly Rigor

No. 238 "A mathematician is a machine for turning coffee into theorems." January 12, 2019

39 Problems in Expanding and Contracting Logarithms

(Part 2)

ANSWERS

1. $\ln(3)+\ln(2)+\ln(11)$	2. $\ln(5)+\ln(3)$
3. $5\ln(3)+5\ln(2)-5\ln(11)$	4. $\ln(3)+4\ln(2)$
5. $4\ln(2)-\ln(5)$	6. $6\ln(3)+6\ln(2)-6\ln(5)$
7. $\ln(x)-6\ln(y)$	8. $2\ln(a)+2\ln(b)$
9. $4\ln(u)-\ln(v)$	10. $\ln(x)-5\ln(y)$
11. $\frac{1}{3}\ln(x)+\frac{1}{3}\ln(y)+\frac{1}{3}\ln(z)$	12. $\ln(x)+\ln(y)+2\ln(z)$
13. $4\ln(x)-2\ln(y)$	14. $3\ln(2)-2\ln(5)$
15. $3\ln(z)+\frac{1}{2}\ln(x)+\frac{1}{2}\ln(y)$	16. $3\ln(a)-3\ln(b)$
17. $2\ln(u)+6\ln(v)$	18. $8\ln(2)+4\ln(3)+8\ln(7)$
19. $\ln(2)+2x\ln(x)$	20. $2\ln(2)+x^4\ln(x)$
21. $3\ln(3x^4+4)+\frac{1}{2}\ln(5x^3+1)$	22. $\frac{1}{3}\ln(4x^2-1)+4\ln(14x^5+7)$
23. $4\ln(x^2+3)-5\ln(5x^5-2)-2\ln(3x^2-5)$	
24. $2\ln(\sin(x))+\ln(\cos(x))-4\ln(x)-3\tan(4x)$	
25. $\frac{1}{2}\ln(\theta)-\ln\left(1+\theta^{\frac{1}{2}}\right)$	26. $x\ln\left(a^{\frac{1}{x}}+b^{\frac{1}{x}}\right)-x\ln(2)$
27. $x\ln(x)\cdot\ln(x)$	28. $\sin(x)\cdot\ln(x)$
29. $\ln\left(\frac{3}{8}\right)$	30. $\ln\left(6^{\frac{1}{3}}\right)$
31. $\ln\left(\frac{3}{8}\right)^4$	32. $\ln(154)$
33. $\ln\left(\frac{7}{12^2}\right)$	34. $\ln\left(7^{\frac{2}{3}}\right)$
35. $\ln(uv)^6$	36. $\ln\left(\frac{x}{y^4}\right)$
37. $\ln\left(\frac{u}{v^6}\right)$	38. $\ln(u^{20}v^5)$
39. $\ln\left(\frac{u^4}{v^{20}}\right)$	

SELECTED SOLUTIONS

1. $\ln(6 \cdot 11) = \ln(6) + \ln(11) = \ln(3 \cdot 2) + \ln(11) = \ln(3) + \ln(2) + \ln(11).$

3. $\ln\left(\frac{6}{11}\right)^5 = 5\ln\left(\frac{6}{11}\right) = 5[\ln(6) - \ln(11)] = 5\ln(3) + 5\ln(2) - 5\ln(11).$

5. $\ln\left(\frac{2^4}{5}\right) = \ln(2^4) - \ln(5) = 4\ln(2) - \ln(5).$

7. $\ln\left(\frac{x}{y^6}\right) = \ln(x) - \ln(y^6) = \ln(x) - 6\ln(y).$

11. $\ln\left(\sqrt[3]{x \cdot y \cdot z}\right) = \ln(x \cdot y \cdot z)^{\frac{1}{3}} = \frac{1}{3}\ln(x \cdot y \cdot z) = \frac{1}{3}[\ln(x) + \ln(y) + \ln(z)] =$
$= \frac{1}{3}\ln(x) + \frac{1}{3}\ln(y) + \frac{1}{3}\ln(z).$

17. $\ln(uv^3)^2 = 2\ln(uv^3) = 2[\ln(u) + \ln(v^3)] = 2[\ln(u) + 3\ln(v)] = 2\ln(u) + 6\ln(v).$

19. $\ln(2x^{2x}) = \ln(2) + \ln(x^{2x}) = \ln(2) + 2x\ln(x).$

21. $\ln(3x^4 + 4)^3\sqrt{5x^3 + 1} = \ln(3x^4 + 4)^3 + \ln\sqrt{5x^3 + 1} = \ln(3x^4 + 4)^3 + \ln(5x^3 + 1)^{\frac{1}{2}} =$
$= 3\ln(3x^4 + 4) + \frac{1}{2}\ln(5x^3 + 1).$

23. $\ln\frac{(x^2+3)^4}{(5x^5-2)^5 \cdot (3x^2-5)^2} = \ln(x^2 + 3)^4 - \ln(5x^5 - 2)^5 \cdot (3x^2 - 5)^2 =$
$= \ln(x^2 + 3)^4 - [\ln(5x^5 - 2)^5 \cdot (3x^2 - 5)^2] =$
$= \ln(x^2 + 3)^4 - [\ln(5x^5 - 2)^5 + \ln(3x^2 - 5)^2] =$
$= \ln(x^2 + 3)^4 - \ln(5x^5 - 2)^5 - \ln(3x^2 - 5)^2 =$
$= 4\ln(x^2 + 3) - 5\ln(5x^5 - 2) - 2\ln(3x^2 - 5).$

25. $\ln\frac{\sqrt{\theta}}{1+\sqrt{\theta}} = \ln\sqrt{\theta} - \ln\left(1 + \sqrt{\theta}\right) = \ln\theta^{\frac{1}{2}} - \ln\left(1 + \theta^{\frac{1}{2}}\right) = \frac{1}{2}\ln(\theta) - \ln\left(1 + \theta^{\frac{1}{2}}\right).$

27. $\ln\left(x^{\ln(x)}\right)^x = x\ln\left(x^{\ln(x)}\right) = x\ln(x) \cdot \ln(x).$

29. $\ln(3) - \ln(8) = \ln\left(\frac{3}{8}\right).$

35. $6\ln(u) + 6\ln(v) = \ln(u^6) + \ln(v^6) = \ln(u^6v^6) = \ln(uv)^6.$

39. $4\ln(u) - 20\ln(v) = \ln(u^4) - \ln(v^{20}) = \ln\left(\frac{u^4}{v^{20}}\right).$