The Weekly Rigor

No. 261 "A mathematician is a machine for turning coffee into theorems." June 22, 2019

10 Problems Concerning the Unit Circle (Part 2 of 2)

(Part 1)

PROBLEMS

Find the values of the six trigonometric functions of the following angles given in radians. Show (write out) the use of reference angles and the reference triangles to determine the values.

1. $\frac{4\pi}{3}$

2. $\frac{7\pi}{4}$

3. $\frac{11\pi}{6}$

4. $\frac{3\pi}{4}$

5. $\frac{5\pi}{3}$

6. $\frac{5\pi}{6}$

7. $\frac{5\pi}{4}$

8. $\frac{2\pi}{3}$

9. $\frac{7\pi}{6}$

10. $\frac{\pi}{3}$

ANSWERS

1. $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$	$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$	$\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$
$\csc\left(\frac{4\pi}{3}\right) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	$\sec\left(\frac{4\pi}{3}\right) = -\frac{2}{1} = -2$	$\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
2. $\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\tan\left(\frac{7\pi}{4}\right) = -\frac{1}{1} = -1$
$\csc\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{1} = -\sqrt{2}$	$\sec\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{1} = \sqrt{2}$	$\cot\left(\frac{7\pi}{4}\right) = -\frac{1}{1} = -1$
3. $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$	$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\tan\left(\frac{11\pi}{6}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
$\csc\left(\frac{11\pi}{6}\right) = -\frac{2}{1} = -2$	$\sec\left(\frac{11\pi}{6}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\cot\left(\frac{11\pi}{6}\right) = -\frac{\sqrt{3}}{1} = -\sqrt{3}$
4. $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\tan\left(\frac{3\pi}{4}\right) = -\frac{1}{1} = -1$
$\csc\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{1} = \sqrt{2}$	$\sec\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{1} = -\sqrt{2}$	$\cot\left(\frac{3\pi}{4}\right) = -\frac{1}{1} = -1$
5. $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$	$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$	$\tan\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{1} = -\sqrt{3}$
$\csc\left(\frac{5\pi}{3}\right) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	$\sec\left(\frac{5\pi}{3}\right) = \frac{2}{1} = 2$	$\cot\left(\frac{5\pi}{3}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
6. $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$	$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$	$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
$\csc\left(\frac{5\pi}{6}\right) = \frac{2}{1} = 2$	$\sec\left(\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	$\cot\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{1} = -\sqrt{3}$
7. $\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\tan\left(\frac{5\pi}{4}\right) = \frac{1}{1} = 1$
$\csc\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{1} = -\sqrt{2}$	$\sec\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{1} = -\sqrt{2}$	$\cot\left(\frac{5\pi}{4}\right) = \frac{1}{1} = 1$
8. $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$	$\tan\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{1} = -\sqrt{3}$
$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\sec\left(\frac{2\pi}{3}\right) = -\frac{2}{1} = -2$	$\cot\left(\frac{2\pi}{3}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
9. $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$	$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$	$\tan\left(\frac{7\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\csc\left(\frac{7\pi}{6}\right) = -\frac{2}{1} = -2$	$\sec\left(\frac{7\pi}{6}\right) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$	$\cot\left(\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{1} = \sqrt{3}$
10. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1} = \sqrt{3}$
$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\sec\left(\frac{\pi}{3}\right) = \frac{2}{1} = -2$	$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$