

The Weekly Rigor

No. 262

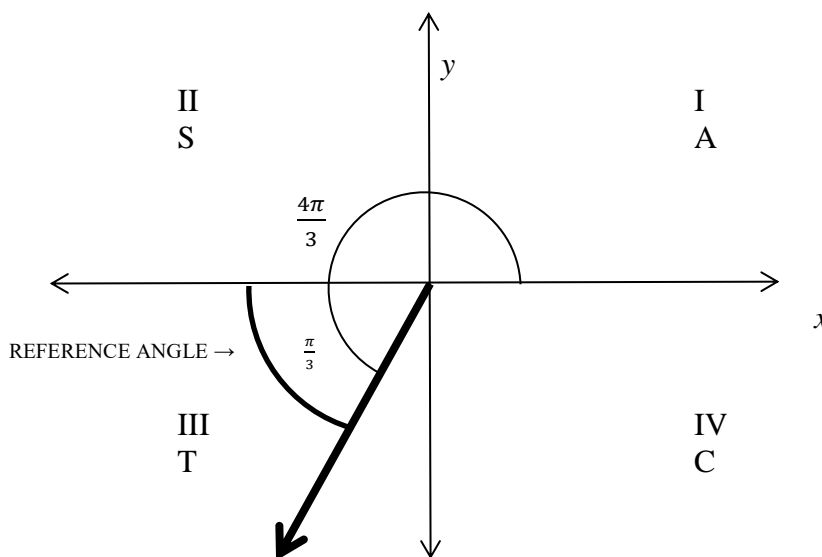
“A mathematician is a machine for turning coffee into theorems.”

June 29, 2019

10 Problems Concerning the Unit Circle (Part 2 of 2) (Part 2)

SELECTED SOLUTIONS

1. $\frac{4\pi}{3}$

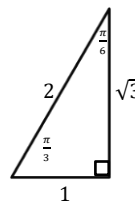


For an angle of $\frac{4\pi}{3}$ in standard position, the reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

The 30-60-90 reference triangle shows us that:

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$$

$$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{\pi}{3}\right) = \frac{2}{1} \quad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

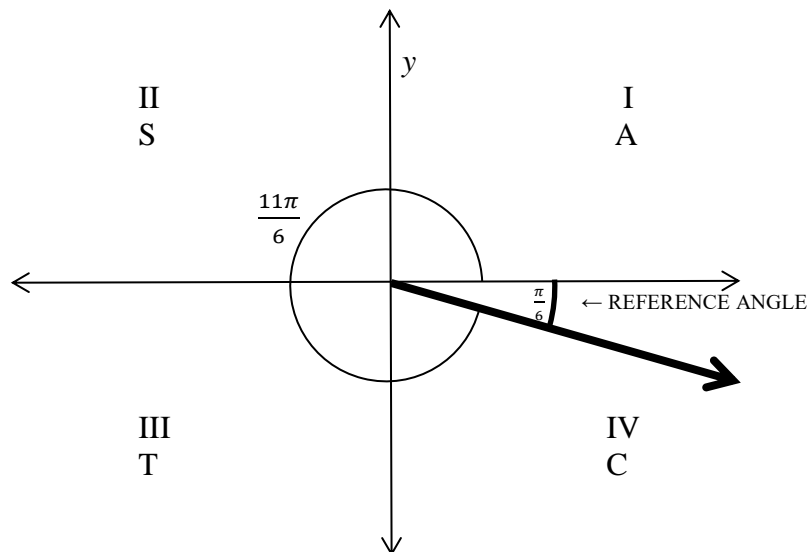


Using All Students Take Calculus and the fact that the angle $\frac{4\pi}{3}$ is in Quadrant III, we get:

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{3}\right) = -\frac{1}{2} \quad \tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$$

$$\csc\left(\frac{\pi}{3}\right) = -\frac{2}{\sqrt{3}} \quad \sec\left(\frac{\pi}{3}\right) = -\frac{2}{1} \quad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

3. $\frac{11\pi}{6}$

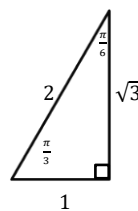


For an angle of $\frac{11\pi}{6}$ in standard position, the reference angle is $2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

The 30-60-90 reference triangle shows us that:

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$\csc\left(\frac{\pi}{6}\right) = \frac{2}{1} \quad \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \quad \cot\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{1}$$



Using All Students Take Calculus and the fact that the angle $\frac{11\pi}{6}$ is in Quadrant IV, we get:

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2} \quad \cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad \tan\left(\frac{11\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$\csc\left(\frac{11\pi}{6}\right) = -\frac{2}{1} \quad \sec\left(\frac{11\pi}{6}\right) = \frac{2}{\sqrt{3}} \quad \cot\left(\frac{11\pi}{6}\right) = -\frac{\sqrt{3}}{1}$$

“Only he who never plays, never loses.”