

The Weekly Rigor

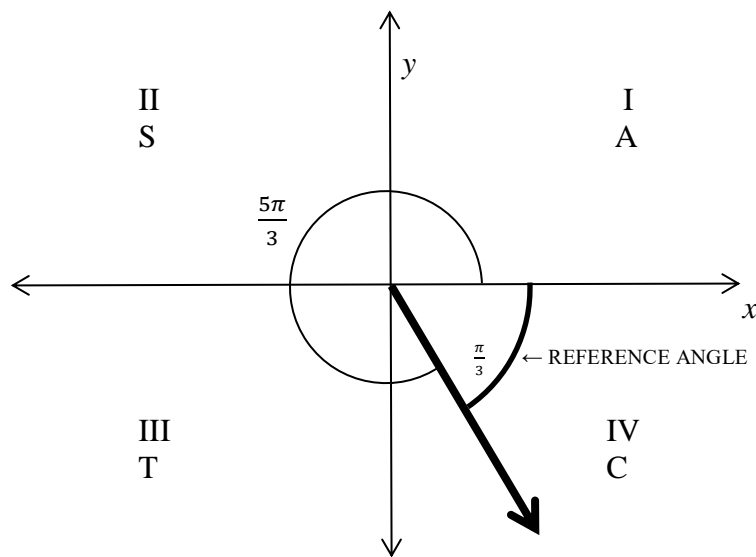
No. 263

“A mathematician is a machine for turning coffee into theorems.”

July 6, 2019

10 Problems Concerning the Unit Circle (Part 2 of 2) (Part 3)

5. $\frac{5\pi}{3}$

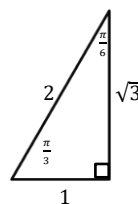


For an angle of $\frac{5\pi}{3}$ in standard position, the reference angle is $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

The 30-60-90 reference triangle shows us that:

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$$

$$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{\pi}{3}\right) = \frac{2}{1} \quad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

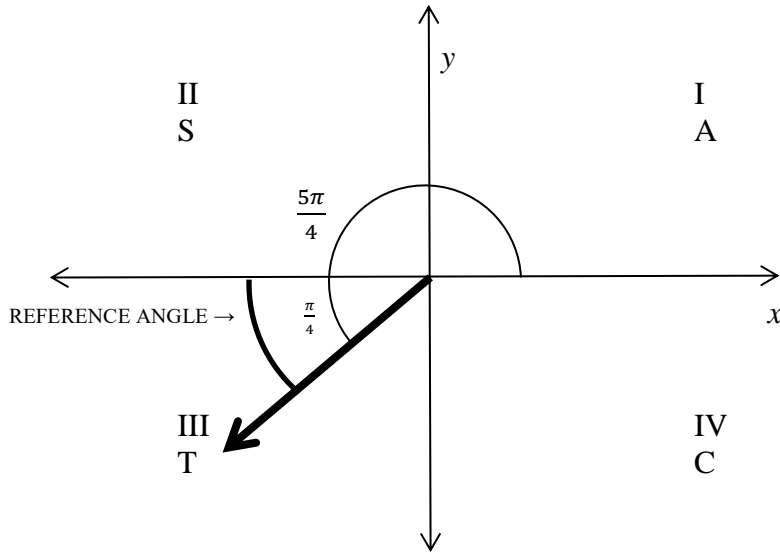


Using All Students Take Calculus and the fact that the angle $\frac{5\pi}{3}$ is in Quadrant IV, we get:

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2} \quad \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \tan\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{1}$$

$$\csc\left(\frac{5\pi}{3}\right) = -\frac{2}{\sqrt{3}} \quad \sec\left(\frac{5\pi}{3}\right) = \frac{2}{1} \quad \cot\left(\frac{5\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$

7. $\frac{5\pi}{4}$

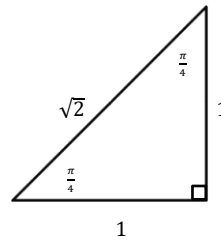


For an angle of $\frac{5\pi}{4}$ in standard position, the reference angle is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

The 45-45-90 reference triangle shows us that:

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \tan\left(\frac{\pi}{4}\right) = \frac{1}{1}$$

$$\csc\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{1} \quad \sec\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{1} \quad \cot\left(\frac{\pi}{4}\right) = \frac{1}{1}$$



Using All Students Take Calculus and the fact that the angle $\frac{5\pi}{4}$ is in Quadrant III, we get:

$$\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} \quad \cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} \quad \tan\left(\frac{5\pi}{4}\right) = \frac{1}{1}$$

$$\csc\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{1} \quad \sec\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{1} \quad \cot\left(\frac{5\pi}{4}\right) = \frac{1}{1}$$

“Only he who never plays, never loses.”