The Weekly Rigor

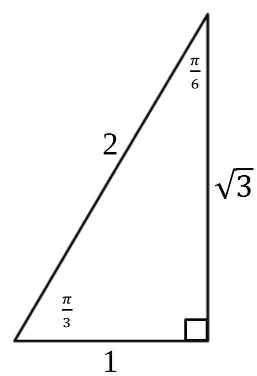
No. 265 | "A mathematician is a machine for turning coffee into theorems." | July 20, 2019

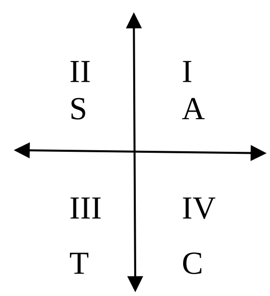
28 Problems Solving Simple Trigonometric Equations (Type I)

(Part 2)

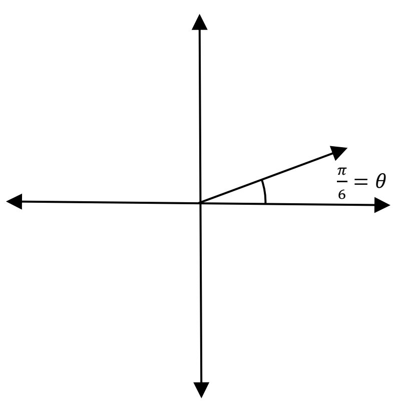
SELECTED SOLUTIONS

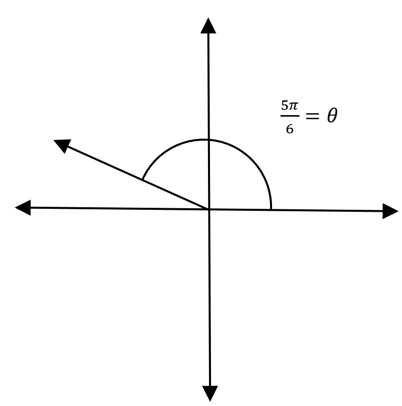
1. $2\sin(\theta) - 1 = 0 \quad \Longrightarrow \quad \sin(\theta) = \frac{1}{2}$. Consulting the 30-60-90 reference triangle,



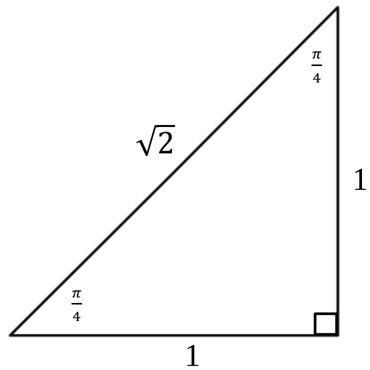


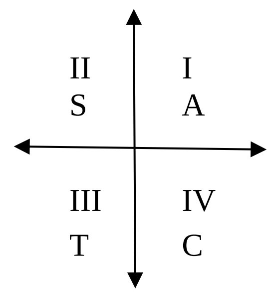
we see that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{6}$. But sine is positive in Quadrants I and II. Therefore, $\theta = \frac{\pi}{6}$ (QI) and $\theta = \pi - \theta_R = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (QII).





7. $\sqrt{2}\sin(\theta) + 1 = 0 \quad \Longrightarrow \quad \sin(\theta) = \frac{-1}{\sqrt{2}}$. Consulting the 45-45-90 reference triangle,





we see that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{4}$.
But sine is negative in Quadrants III and IV. Therefore,
$\theta = \pi + \theta_R = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ (QIII) and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ (QIV).