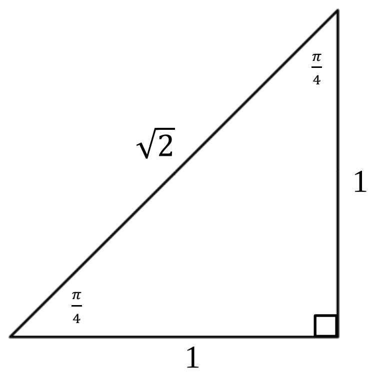
The Weekly Rigor

No. 266 “A mathematician is a machine for turning coffee into theorems.” July 27, 2019

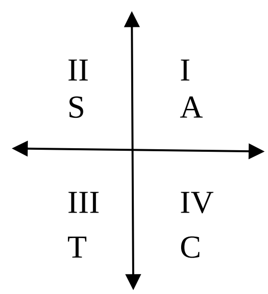
28 Problems Solving Simple Trigonometric Equations (Type I)

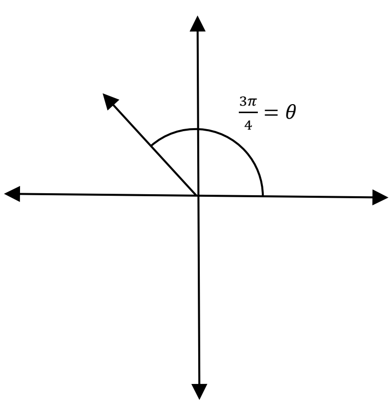
(Part 3)

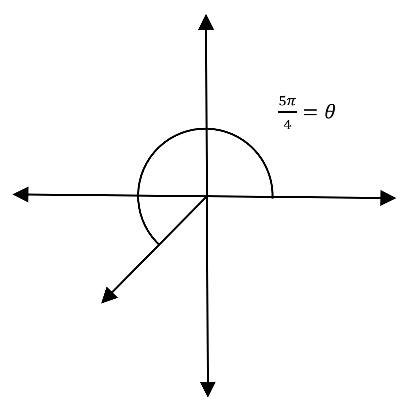
9. $2\cos(\theta) + \sqrt{2} = 0 \quad \Longrightarrow \quad \cos(\theta) = \frac{-\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$. Consulting the 45-45-90 reference triangle,



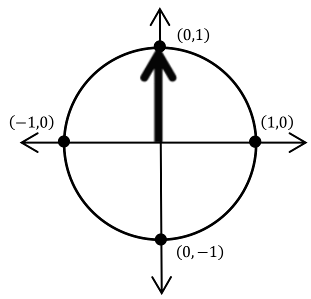
we see that $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{4}$. But cosine is negative in Quadrants II and III. Therefore, $\theta = \pi - \theta_R = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ (QII) and $\theta = \pi + \theta_R = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ (QIII).





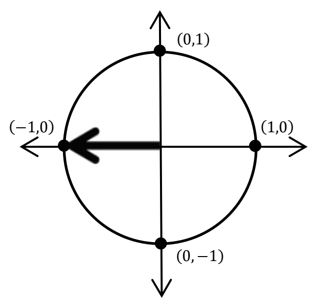


13. $\sin(\theta) - 1 = 0 \quad \Longrightarrow \quad \sin(\theta) = 1.$



For any angle θ in standard position and its corresponding point (x, y) on the unit circle, $(\cos(\theta), \sin(\theta)) = (x, y)$. By inspection, $\left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right)\right) = (0,1)$, i.e., $\sin\left(\frac{\pi}{2}\right) = 1$. Therefore, $\theta = \frac{\pi}{2}$.

19. $\cos(\theta) + 1 = 0 \quad \Longrightarrow \quad \cos(\theta) = -1.$



For any angle θ in standard position and its corresponding point (x, y) on the unit circle, $(\cos(\theta), \sin(\theta)) = (x, y)$. By inspection, $(\cos(\pi), \sin(\pi)) = (-1,0)$, i.e., $\cos(\pi) = -1$. Therefore, $\theta = \pi$.