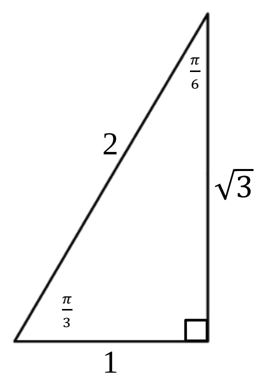
The Weekly Rigor

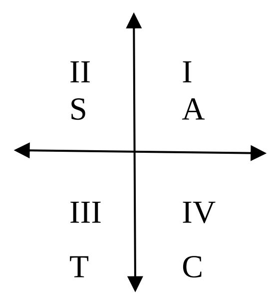
No. 267 “A mathematician is a machine for turning coffee into theorems.” August 3, 2019

28 Problems Solving Simple Trigonometric Equations (Type I)

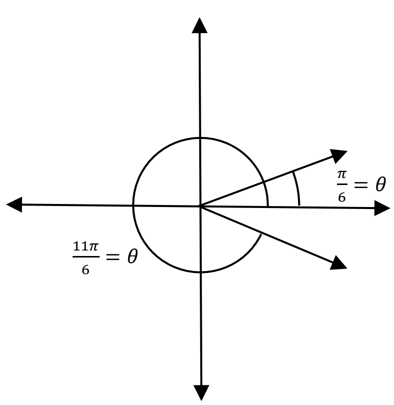
(Part 4)

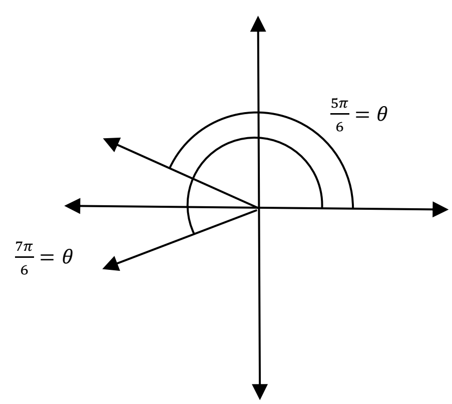
25. $4\cos^2(\theta) - 3 = 0 \quad \Longrightarrow \quad \cos^2(\theta) = \frac{3}{4} \quad \Longrightarrow \quad \cos(\theta) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$.
Consulting the 30-60-90 reference triangle,





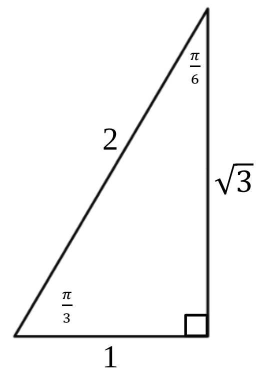
we see that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{6}$. But cosine is positive in Quadrants I and IV. Therefore, $\theta = \frac{\pi}{6}$ (QI) and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ (QIV). Furthermore, cosine is negative in Quadrants II and III. Therefore, $\theta = \pi - \theta_R = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (QII) and $\theta = \pi + \theta_R = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ (QIII).

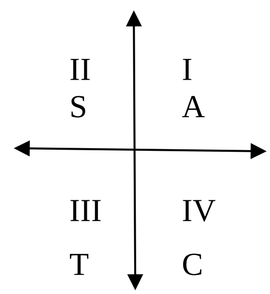




26. $4\sin^2(\theta) - 3 = 0 \quad \Longrightarrow \quad \sin^2(\theta) = \frac{3}{4} \quad \Longrightarrow \quad \sin(\theta) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

Consulting the 30-60-90 reference triangle,





we see that $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{3}$. But sine is positive in Quadrants I and II. Therefore, $\theta = \frac{\pi}{3}$ (QI) and $\theta = \pi - \theta_R = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (QII). Furthermore, sine is negative in Quadrants III and IV. Therefore, $\theta = \pi + \theta_R = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ (QIII) and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ (QIV).