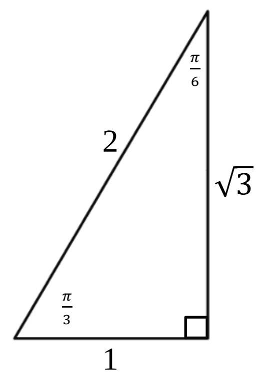
The Weekly Rigor

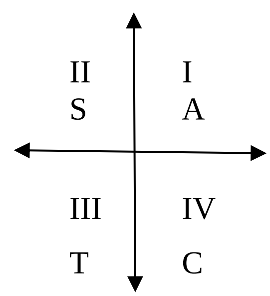
No. 268 | "A mathematician is a machine for turning coffee into theorems." | August 10, 2019

28 Problems Solving Simple Trigonometric Equations (Type I)

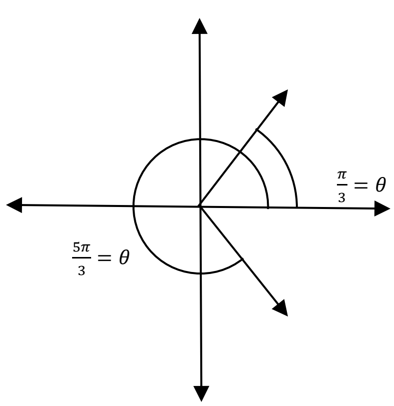
(Part 5)

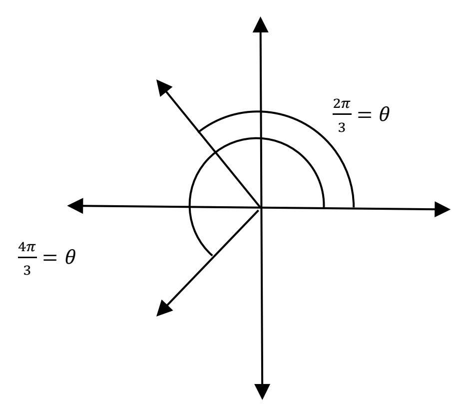
27. $4\cos^2(\theta) - 1 = 0 \quad \Longrightarrow \quad \cos^2(\theta) = \frac{1}{4} \quad \Longrightarrow \quad \cos(\theta) = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$.
Consulting the 30-60-90 reference triangle,



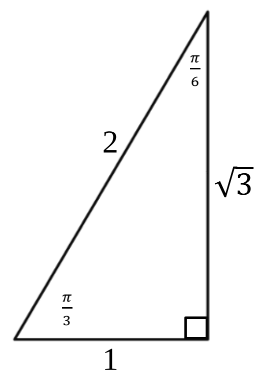


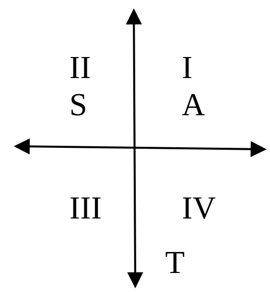
we see that $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{3}$. But cosine is positive in Quadrants I and IV. Therefore, $\theta = \frac{\pi}{3}$ (QI) and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ (QIV). Furthermore, cosine is negative in Quadrants II and III. Therefore, $\theta = \pi - \theta_R = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (QII) and $\theta = \pi + \theta_R = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ (QIII).





28. $2\cos(\theta) - \sqrt{3} = 0 \quad \Longrightarrow \quad \cos(\theta) = \frac{\sqrt{3}}{2}$. Consulting the 30-60-90 reference triangle,





we see that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{6}$. But cosine is positive in Quadrants I and IV. Therefore, $\theta = \frac{\pi}{6}$ (QI) and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ (QIV).

C