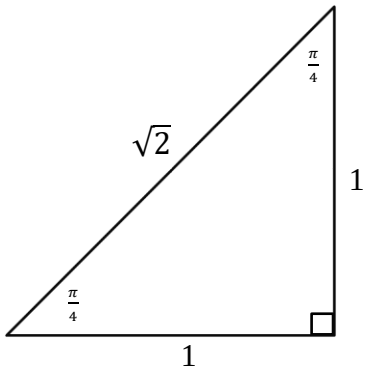
The Weekly Rigor

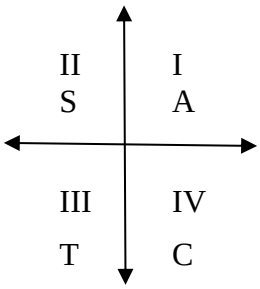
No. 273 "A mathematician is a machine for turning coffee into theorems." September 14, 2019

28 Problems Solving Simple Trigonometric Equations (Type III)

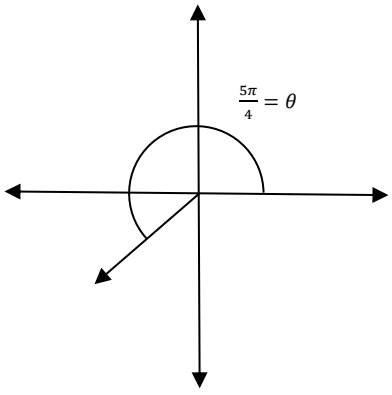
(Part 3)

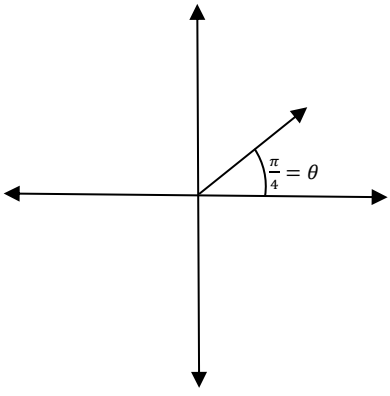
21. $\tan(\theta) - 1 = 0 \implies \tan(\theta) = 1$. Consulting the 45-45-90 reference triangle,



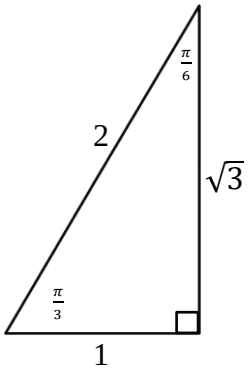


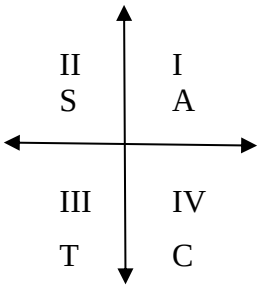
we see that $\tan\left(\frac{\pi}{4}\right) = \frac{1}{1} = 1$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{4}$. But tangent is positive in Quadrants I and III. Therefore, $\theta = \frac{\pi}{4}$ (QI) and $\theta = \pi + \theta_R = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ (QIII).





26. $\cot^2(\theta) - 3 = 0 \implies \cot^2(\theta) = 3 \implies \frac{1}{\tan^2(\theta)} = 3 \implies \tan^2(\theta) = \frac{1}{3} \implies$
$\implies \tan(\theta) = \pm\frac{1}{\sqrt{3}}$. Consulting the 30-60-90 reference triangle,





we see that $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$. Hence, θ_R, the reference angle for θ, is $\frac{\pi}{6}$.
But tangent is positive in Quadrants I and III. Therefore, $\theta = \frac{\pi}{6}$ (QI)
and $\theta = \pi + \theta_R = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ (QII). Furthermore, tangent is
negative in Quadrants II and IV. Therefore, $\theta = \pi - \theta_R = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ (QII)
and $\theta = 2\pi - \theta_R = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ (QIV).