The Weekly Rigor

No. 276

"A mathematician is a machine for turning coffee into theorems."

October 5, 2019

12 Problems Solving Composite Trigonometric Equations (Type II) (Part 1)

Type II Equations: Involving secant or cosecant.

PROBLEMS

Solve for x over the interval $[0,2\pi)$. Show (write out) the use of reference angles and the reference triangles to determine the solution(s), except in cases where x is a quadrant angle $(0, \frac{\pi}{2}, \pi, \text{ and } \frac{3\pi}{2})$.

1.
$$\csc\left(\frac{x}{3}\right) - 2 = 0$$
 2. $3 \sec\left(\frac{x}{2}\right) - 2\sqrt{3} = 0$

3.
$$\sqrt{3} \sec(2x) - 2 = 0$$

4. $\csc\left(\frac{1}{4}x\right) + 2 = 0$

5. $3 \sec(2x) + 2\sqrt{3} = 0$ 6. $\csc(3x) + \sqrt{2} = 0$

7.
$$\sqrt{3}\sec\left(\frac{2}{3}x\right) + 2 = 0$$
 8. $\sec(5x) + 1 = 0$

9.
$$\csc\left(\frac{x}{2}\right) - 1 = 0$$
 10. $\sec\left(\frac{x}{2}\right) - 1 = 0$

11.
$$3\csc(2x) + 2\sqrt{3} = 0$$
 12. $\sqrt{3}\csc\left(\frac{3}{2}x\right) + 2 = 0$

ANSWERS

1. $\frac{\pi}{2}$	2. $\frac{\pi}{3}$	3. $\frac{\pi}{12}$, $\frac{11\pi}{12}$, $\frac{13\pi}{12}$, $\frac{23\pi}{12}$	4. No solution
5. $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$	$6. \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{21\pi}{12}, \\ \frac{23\pi}{12}, \frac{15\pi}{12}, \frac{5\pi}{12}, \frac{5\pi}{$	7. $\frac{5\pi}{4}, \frac{7\pi}{4}$	8. $\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$
9. π	10. 0	11. $\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$	12. $\frac{8\pi}{9}, \frac{10\pi}{9}$

"Only he who never plays, never loses."