

The Weekly Rigor

No. 283

“A mathematician is a machine for turning coffee into theorems.”

November 23, 2019

39 Problems in Factoring by the Sum or Difference of Cubes (Part 2)

ANSWERS

1. $(x + 4)(x^2 - 4x + 16)$	2. $(x - 4)(x^2 + 4x + 16)$	3. $(x - 1)(x^2 + x + 1)$
4. $(x + 2)(x^2 - 2x + 4)$	5. $(x + 3)(x^2 - 3x + 9)$	6. $(x - 4)(x^2 + 4x + 16)$
7. $(2 - x)(4 + 2x + x^2)$	8. $(1 + x)(1 - x + x^2)$	9. $(4 + x)(16 - 4x + x^2)$
10. $(2x - 3)(4x^2 + 6x + 9)$	11. $(3x - 1)(9x^2 + 3x + 1)$	12. $(4x + 3)(16x^2 - 12x + 9)$
13. $(3 + 2x)(9 - 6x + 4x^2)$	14. $4(2 - x)(4 + 2x + x^2)$	15. $(1 - 2x)(1 + 2x + 4x^2)$
16. $(2x + y)(4x^2 - 2xy + y^2)$	17. $(4x + y)(16x^2 - 4xy + y^2)$	18. $(5x - y)(25x^2 + 5xy + y^2)$
19. $(x - 2y)(x^2 + 2xy + 4y^2)$	20. $(x + 3y)(x^2 - 3xy + 9y^2)$	21. $(x + 5y)(x^2 - 5xy + 25y^2)$
22. $(2x - 3y)(4x^2 + 6xy + 9y^2)$	23. $(4x - 3y)(16x^2 + 12xy + 9y^2)$	
24. $(5x + 4y)(25x^2 - 20xy + 16y^2)$	25. $(x + y + 2)(x^2 + y^2 - 2x - 2y + 2xy + 4)$	
26. $(x - y - 4)(x^2 + y^2 + 4x - 4y - 2xy + 16)$	27. $(2 - x - y)(x^2 + y^2 + 2x + 2y + 2xy + 4)$	
28. $(4 + x - y)(x^2 + y^2 - 4x + 4y - 2xy + 16)$	29. $(2x + 2y + 3)(4x^2 + 4y^2 - 6x - 6y + 8xy + 9)$	
30. $(4x - 4y - 1)(16x^2 - 16y^2 + 4x - 4y - 32xy + 1)$	31. $-2(3x^2 + 24x + 49)$	
32. $(2x + 1)(x^2 + x + 7)$	33. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$	
34. $(x^2 - y^2)(x^4 + x^2y^2 + y^4)$	35. $27(x - 3)(x^2 - 3x + 3)$	
36. $(2x - 3)(4x^2 - 24x + 39)$	37. $(\cos(\theta) + \sin(\theta))(1 - \cos(\theta) \sin(\theta))$	
38. $(\cos(\theta) - \sin(\theta))(1 + \cos(\theta) \sin(\theta))$	39. $(e^x - \ln(x))(e^{2x} + e^x \ln(x) + \ln^2(x))$	

SELECTED SOLUTIONS

Remark: For these type of factoring problems, keep in mind the “template” patterns of $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

1. $x^3 + 4^3 = (x + 4)(x^2 - 4x + 4^2) = (x + 4)(x^2 - 4x + 16)$.

3. $x^3 - 1^3 = (x - 1)(x^2 + 1x + 1^2) = (x - 1)(x^2 + 1x + 1)$.

5. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 3^2) = (x + 3)(x^2 - 3x + 9)$.

7. $8 - x^3 = 2^3 - x^3 = (2 - x)(2^2 + 2x + x^2) = (2 - x)(4 + 2x + x^2)$.

9. $64 + x^3 = 4^3 + x^3 = (4 + x)(4^2 - 4x + x^2) = (4 + x)(16 - 4x + x^2)$.

11. $27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)((3x)^2 + 3x + 1^2) = (3x - 1)(9x^2 + 3x + 1)$.
13. $27 + 8x^3 = 3^3 + (2x)^3 = (3 + 2x)(3^2 - 6x + (2x)^2) = (3 + 2x)(9 - 6x + 4x^2)$.
15. $1 - 8x^3 = 1^3 - (2x)^3 = (1 - 2x)(1^2 + 2x + (2x)^2) = (1 - 2x)(1 + 2x + 4x^2)$.
17. $64x^3 + y^3 = (4x)^3 + y^3 = (4x + y)((4x)^2 - 4xy + y^2) = (4x + y)(16x^2 - 4xy + y^2)$.
19. $x^3 - 8y^3 = x^3 - (2y)^3 = (x - 2y)(x^2 + 2xy + (2y)^2) = (x - 2y)(x^2 + 2xy + 4y^2)$.
21. $x^3 + 125y^3 = x^3 + (5y)^3 = (x + 5y)(x^2 - 5xy + (5y)^2) = (x + 5y)(x^2 - 5xy + 25y^2)$.
23. $64x^3 - 27y^3 = (4x)^3 - (3y)^3 = (4x - 3y)((4x)^2 + 12xy + (3y)^2) = (4x - 3y)(16x^2 + 12xy + 9y^2)$.
25. $(x + y)^3 + 8 = (x + y)^3 + 2^3 = ((x + y) + 2)((x + y)^2 - 2(x + y) + 2^2) = (x + y + 2)(x^2 + 2xy + y^2 - 2x - 2y + 4) = (x + y + 2)(x^2 + y^2 - 2x - 2y + 2xy + 4)$.
27. $8 - (x + y)^3 = 2^3 - (x + y)^3 = (2 - (x + y))(2^2 + 2(x + y) + (x + y)^2) = (2 - x - y)(4 + 2x + 2y + x^2 + 2xy + y^2) = (2 - x - y)(x^2 + y^2 + 2x + 2y + 2xy + 4)$.
29. $8(x + y)^3 + 27 = [2(x + y)]^3 + 3^3 = [2(x + y) + 3] \left[(2(x + y))^2 - 6(x + y) + 3^2 \right] = (2x + 2y + 3)[4(x^2 + 2xy + y^2) - 6x - 6y + 9] = (2x + 2y + 3)(4x^2 + 8xy + 4y^2 - 6x - 6y + 9) = (2x + 2y + 3)(4x^2 + 4y^2 - 6x - 6y + 8xy + 9)$.
31. $(x + 3)^3 - (x + 5)^3 = [(x + 3) - (x + 5)][(x + 3)^2 + (x + 3)(x + 5) + (x + 5)^2] = (x + 3 - x - 5)(x^2 + 6x + 9 + x^2 + 8x + 15 + x^2 + 10x + 25) = -2(3x^2 + 24x + 49)$.
33. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)((x^2)^2 - x^2y^2 + (y^2)^2) = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.
35. $(3x - 6)^3 - 27 = (3x - 6)^3 - 3^3 = [(3x - 6) - 3][(3x - 6)^2 + 3(3x - 6) + 3^2] = (3x - 6 - 3)(9x^2 - 36x + 36 + 9x - 18 + 9) = (3x - 9)(9x^2 - 27x + 27) = 3(x - 3)9(x^2 - 3x + 3) = 27(x - 3)(x^2 - 3x + 3)$.
37. $\cos^3(\theta) + \sin^3(\theta) = (\cos(\theta) + \sin(\theta))(\cos^2(\theta) - \cos(\theta)\sin(\theta) + \sin^2(\theta)) = (\cos(\theta) + \sin(\theta))(\sin^2(\theta) + \cos^2(\theta) - \cos(\theta)\sin(\theta)) = (\cos(\theta) + \sin(\theta))(1 - \cos(\theta)\sin(\theta))$.
39. $e^{3x} - \ln^3(x) = (e^x - \ln(x))(e^{2x} + e^x \ln(x) + \ln^2(x))$.

“Only he who never plays, never loses.”