

The Weekly Rigor

No. 287

“A mathematician is a machine for turning coffee into theorems.”

December 21, 2019

60 Problems in Factoring by a Mixture of Methods (Part 4)

SELECTED SOLUTIONS

1. $x^3 + 125y^3 = x^3 + (5y)^3 = (x + 5y)(x^2 - 5xy + (5y)^2) = (x + 5y)(x^2 - 5xy + 25y^2)$.

3. $8 - (x + y)^3 = 2^3 - (x + y)^3 = (2 - (x + y))(2^2 + 2(x + y) + (x + y)^2) =$
 $= (2 - x - y)(4 + 2x + 2y + x^2 + 2xy + y^2) = (2 - x - y)(x^2 + y^2 + 2x + 2y + 2xy + 4)$.

5.

36	
1	36
2	18
3	12
4	9
6	6

$2x^2 + 15x + 18 = 2x^2 + 3x + 12x + 18 = x(2x + 3) + 6(2x + 3) = (2x + 3)(x + 6)$.

7. $m^3 + m = m \cdot m^2 + m = m(m^2 + 1)$.

9. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)((x^2)^2 - x^2y^2 + (y^2)^2) = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.

11. $x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3) = (x + 3)(x^2 - 4) =$
 $= (x + 3)(x + 2)(x - 2)$.

13. $x^3 + 4^3 = (x + 4)(x^2 - 4x + 4^2) = (x + 4)(x^2 - 4x + 16)$.

15. $3x^2 - 75 = 3(x^2 - 25) = 3(x^2 - 5^2) = 3(x + 5)(x - 5)$.

$$\begin{aligned}
 17. (3x - 6)^3 - 27 &= (3x - 6)^3 - 3^3 = [(3x - 6) - 3][(3x - 6)^2 + 3(3x - 6) + 3^2] = \\
 &= (3x - 6 - 3)(9x^2 - 36x + 36 + 9x - 18 + 9) = (3x - 9)(9x^2 - 27x + 27) = \\
 &= 3(x - 3)9(x^2 - 3x + 3) = 27(x - 3)(x^2 - 3x + 3).
 \end{aligned}$$

19.

-6	
-1	6
1	-6
-2	3
2	-3

$$\begin{aligned}
 12x^3 - 4x^2 - 8x &= 4x(3x^2 - x - 2) = 4x(3x^2 - 3x + 2x - 2) = \\
 &= 4x[3x(x - 1) + 2(x - 1)] = 4x(3x + 2)(x - 1).
 \end{aligned}$$

$$\begin{aligned}
 21. (x + 3)^3 - (x + 5)^3 &= [(x + 3) - (x + 5)][(x + 3)^2 + (x + 3)(x + 5) + (x + 5)^2] = \\
 &= (x + 3 - x - 5)(x^2 + 6x + 9 + x^2 + 8x + 15 + x^2 + 10x + 25) = -2(3x^2 + 24x + 49).
 \end{aligned}$$

$$\begin{aligned}
 23. (x + y)^3 + 8 &= (x + y)^3 + 2^3 = ((x + y) + 2)((x + y)^2 - 2(x + y) + 2^2) = \\
 &= (x + y + 2)(x^2 + 2xy + y^2 - 2x - 2y + 4) = (x + y + 2)(x^2 + y^2 - 2x - 2y + 2xy + 4).
 \end{aligned}$$

$$\begin{aligned}
 25. 64x^3 - 27y^3 &= (4x)^3 - (3y)^3 = (4x - 3y)((4x)^2 + 12xy + (3y)^2) = \\
 &= (4x - 3y)(16x^2 + 12xy + 9y^2).
 \end{aligned}$$

$$\begin{aligned}
 27. 10mn + 50m + 6n^3 + 30n^2 &= 10m(n + 5) + 6n^2(n + 5) = (10m + 6n^2)(n + 5) = \\
 &= 2(5m + 3n^2)(n + 5).
 \end{aligned}$$

$$29. 64x^3 + y^3 = (4x)^3 + y^3 = (4x + y)((4x)^2 - 4xy + y^2) = (4x + y)(16x^2 - 4xy + y^2).$$

“Only he who never plays, never loses.”