

The Weekly Rigor

No. 288

“A mathematician is a machine for turning coffee into theorems.”

December 28, 2019

60 Problems in Factoring by a Mixture of Methods (Part 5)

$$31. 7xy^2 - 28x^5 + y^2 - 4x^4 = 7x(y^2 - 4x^4) + (y^2 - 4x^4) = (7x + 1)(y^2 - 4x^4) = (7x + 1)(y + 2x^2)(y - 2x^2).$$

$$33. 27 + 8x^3 = 3^3 + (2x)^3 = (3 + 2x)(3^2 - 6x + (2x)^2) = (3 + 2x)(9 - 6x + 4x^2).$$

$$35. 27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)((3x)^2 + 3x + 1^2) = (3x - 1)(9x^2 + 3x + 1).$$

37.

6	
1	6
2	3

$$3x^2 + 5xy + 2y^2 = 3x^2 + 2xy + 3xy + 2y^2 = x(3x + 2y) + y(3x + 2y) = (3x + 2y)(x + y).$$

$$39. 64 + x^3 = 4^3 + x^3 = (4 + x)(4^2 - 4x + x^2) = (4 + x)(16 - 4x + x^2).$$

$$41. e^{3x} - \ln^3(x) = (e^x - \ln(x))(e^{2x} + e^x \ln(x) + \ln^2(x)).$$

$$43. 8 - x^3 = 2^3 - x^3 = (2 - x)(2^2 + 2x + x^2) = (2 - x)(4 + 2x + x^2).$$

$$45. x^3 + 27 = x^3 + 3^2 = (x + 3)(x^2 - 3x + 3^2) = (x + 3)(x^2 - 3x + 9).$$

$$47. 18e^{3x} - 33e^{2x} + 12e^x = 3e^x(6e^{2x} - 11e^x + 4) = 3e^x(6e^{2x} - 3e^x - 8e^x + 4) = 3e^x[3e^x(2e^x - 1) - 4(2e^x - 1)] = 3e^x(2e^x - 1)(3e^x - 4).$$

49.

9	
1	9
-1	-9
3	3
-3	-3

$$9x^2 - 6x + 1 = 9x^2 - 3x - 3x + 1 = 3x(3x - 1) - (3x - 1) = (3x - 1)(3x - 1) = (3x - 1)^2.$$

$$51. \ln^3(x) + 8 = [\ln(x)]^3 + 2^3 = (\ln(x) + 2)([\ln(x)]^2 - 2\ln(x) + 2^2) = (\ln(x) + 2)(\ln^2(x) - 2\ln(x) + 4).$$

$$53. \sin^2(\theta) - 1 = (\sin(\theta))^2 - 1 = (\sin(\theta) + 1)(\sin(\theta) - 1).$$

$$55. e^{2x} - 16 = (e^x)^2 - 4^2 = (e^x + 4)(e^x - 4).$$

$$57. e^{4x} - e^{2x} = e^{2x+2x} - e^{2x} = e^{2x} \cdot e^{2x} - e^{2x} = e^{2x}(e^{2x} - 1) = e^{2x}[(e^x)^2 - 1^2] = e^{2x}(e^x + 1)(e^x - 1).$$

$$59. e^{2x} \sin^2(x) - x^2 \ln^2(x) = [e^x \sin(x)]^2 - [x \ln(x)]^2 = [e^x \sin(x) + x \ln(x)][e^x \sin(x) - x \ln(x)].$$

“Only he who never plays, never loses.”