The Weekly Rigor

No. 341 "A mathematician is a machine for turning coffee into theorems." January 2, 2021

20 Problems in Dividing a Polynomial by a Polynomial

(Part 6)

ANSWERS

1. $a+b$	2. a^2+ab+b^2
3. a^2-ax+x^2	4. x^2+xy+y^2
5. $a-x$	6. $a-3x$
7. m^2-n^2	8. $a^2-2ab+b^2$
9. a^2+ab+b^2	10. x^2-6x+9
11. $m-n$	12. $4x^2+6xy+9y^2$
13. $x+y+z$	14. $9x^2+12xy+16y^2$
15. $3x-1$	16. $4x+3$
17. $3x-8+\frac{9}{x+2}$	18. $2x+1-\frac{3}{x-1}$
19. $2x^2-4x+7-\frac{20}{x+2}$	20. $x^3+x^2+2x-4-\frac{16}{x-2}$

SELECTED SOLUTIONS

1.
$$\begin{array}{r} a\ +\ b \\ \underline{a+b}\,\overline{)\,a^2+2ab+b^2} \\ \underline{a^2+\ ab\ \ \ } \\ ab+b^2 \\ \underline{ab+b^2} \\ 0 \end{array}$$

Check: $(a+b)(a+b)=a^2+2ab+b^2$. ✓

7.
$$\begin{array}{r} m^2-n^2 \\ \underline{m+2n}\,\overline{)\,m^3+2m^2n-mn^2-2n^3} \\ \underline{m^3+2m^2n\ \ \ \ \ \ \ \ } \\ -mn^2-2n^3 \\ \underline{-mn^2-2n^3} \\ 0 \end{array}$$

Check: $(m+2n)(m^2-\ n^2)=$
$=m^3-mn^2+2m^2n-2n^3=$
$=m^3+2m^2n-mn^2-2n^3$. ✓

15.
$$\begin{array}{r} 3x-1 \\ x+4\,\overline{)\,3x^2+11x-4} \\ \underline{3x^2+12x^3} \\ -x-4 \\ \underline{-x-4} \\ 0 \end{array}$$

Check: $(x+4)(3x-1)=$
$=3x^2-x+12x-4=3x^2+11x-4.$ ✓

20.
$$\begin{array}{r} x^3+x^2+2x-4-\frac{16}{x-2} \\ x-2\,\overline{)\,x^4-x^3-8x-8} \\ \underline{x^4-2x^3} \\ x^3 \\ \underline{x^3-2x^2} \\ 2x^2-8x \\ \underline{2x^2-4x} \\ -4x-8 \\ \underline{-4x+8} \\ -16 \end{array}$$

Check: $(x-2)\left(x^3+x^2+2x-4-\frac{16}{x-2}\right)=$
$=x^3(x-2)+x^2(x-2)+2x(x-2)-$
$-4(x-2)-\frac{16}{x-2}(x-2)=$
$=x^4-2x^3+x^3-2x^2+2x^2-4x-4x+$
$+8-16=$
$=x^4-x^3-8x-8.$ ✓