The Weekly Rigor

No. 347 "A mathematician is a machine for turning coffee into theorems." February 13, 2021

40 Problems in Dividing a Polynomial by a Polynomial Using Synthetic Division

(Part 6)

ANSWERS

1. $x+3$	2. $3x+4$
3. $3x-4$	4. $3x-4$
5. $3x+4+\frac{1}{x-2}$	6. $3x+6$
7. $3x+6+\frac{2}{x-2}$	8. $x+6$
9. $x-6$	10. x^2-1
11. x^2-6x+8	12. x^2+x-1
13. x^2-x+1	14. x^2-6x+9
15. $x^2-6x+9+\frac{1}{x-3}$	16. $x^2-6x+8-\frac{3}{x-3}$
17. $3x-1$	18. $4x+3$
19. $3x-8+\frac{9}{x+2}$	20. $2x+1-\frac{3}{x-1}$
21. $2x^2-4x+7-\frac{20}{x+2}$	22. $x^3+x^2+2x-4-\frac{16}{x-2}$
23. $x^4+3x^3+3^2x^2+3^3x+3^4$	24. $x^4-3x^3+3^2x^2-3^3x+3^4$
25. $x+c$	26. $x+c+\frac{2c^2}{x-c}$
27. $x-c$	28. $x-c+\frac{2c^2}{x+c}$
29. x^2+cx+c^2	30. $x^2+cx+c^2+\frac{2c^3}{x-c}$
31. $x^2-cx+c^2-\frac{2c^3}{x-c}$	32. x^2-cx+c^2
33. $x^3+cx^2+c^2x+c^3$	34. $x^3-cx^2+c^2x-c^3$
35. $x^4+cx^3+c^2x^2+c^3x+c^4$	36. $x^4-cx^3+c^2x^2-c^3x+c^4$
37. $x^5+cx^4+c^2x^3+c^3x^2+c^4x+c^5$	38. $x^5-cx^4+c^2x^3-c^3x^2+c^4x-c^5$
39. $x^6+cx^5+c^2x^4+c^3x^3+c^4x^2+c^5x+c^6$	40. $x^6-cx^5+c^2x^4-c^3x^3+c^4x^2-c^5x+c^6$

SELECTED SOLUTIONS

1. $\begin{array}{r|rrr} 2 & 1 & 1 & -6 \\ & & 2 & 6 \\ \hline & 1 & 3 & 0 \end{array}$ $x+3$

Check: $(x-2)(x+3)=x(x-2)+3(x-2)=x^2-2x+3x-6=x^2+x-6.$ ✓

5. $\begin{array}{r|rrr} 2 & 3 & -2 & -7 \\ & & 6 & 8 \\ \hline & 3 & 4 & 1 \end{array}$ $3x+4+\frac{1}{x-2}$

Check: $(x-2)\left(3x+4+\frac{1}{x-2}\right)=3x(x-2)+4(x-2)+\left(\frac{1}{x-2}\right)(x-2)=$
$=3x^2-6x+4x-8+1=3x^2-2x-7.$ ✓

11. $\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$ x^2-6x+8

Check: $(x+1)(x^2-6x+8)=x^2(x+1)-6x(x+1)+8(x+1)=$
$=x^3+x^2-6x^2-6x+8x+8=x^3-5x^2+2x+8.$ ✓

25. $\begin{array}{r|rrr} c & 1 & 0 & -c^2 \\ & & c & c^2 \\ \hline & 1 & c & 0 \end{array}$ $x+c$

Check: $(x-c)(x+c)=x(x-c)+c(x-c)=x^2-cx+cx-c^2=x^2-c^2.$ ✓

31. $\begin{array}{r|rrrr} -c & 1 & 0 & 0 & -c^3 \\ & & -c & c^2 & -c^3 \\ \hline & 1 & -c & c^2 & -2c^3 \end{array}$ $x^2-cx+c^2-\frac{2c^3}{x-c}$

Check: $(x+c)\left(x^2-cx+c^2-\frac{2c^3}{x-c}\right)=$
$=x^2(x+c)-cx(x+c)+c^2(x+c)-\left(\frac{2c^3}{x-c}\right)(x+c)=$
$=x^3+cx^2-cx^2-c^2x+c^2x+c^3-2c^3=x^3-c^3.$ ✓