The Weekly Rigor

No. 350 "A mathematician is a machine for turning coffee into theorems." March 6, 2021

20 Problems in Solving Fractional Equations

(Part 3)

17. $3+\frac{6}{t-3}=\frac{6}{t^2-3t}$.

18. $2+\frac{4}{t-1}=\frac{4}{t^2-t}$.

19. $\frac{a}{a-5}+\frac{2}{a-6}=\frac{2}{a^2-11a+30}$.

20. $\frac{a}{a+2}+\frac{3}{a+4}=\frac{14}{a^2+6a+8}$.

ANSWERS

1. 1	2. -4	3. $\frac{3}{4}$	4. $\frac{7}{10}$	5. $\frac{1}{6}, 6$
6. $-\frac{1}{3}, 9$	7. 11	8. $-\frac{4}{27}$	9. 4	10. 2
11. $2, -3$	12. $1, -4$	13. $4, 7$	14. $\frac{-1}{3}$	15. -21
16. -8	17. $-1, 2$	18. -2	19. -2	20. $-8, 1$

SELECTED SOLUTIONS

1. $\frac{x+2}{5} + \frac{x-1}{6} = \frac{3}{5} \quad \Longrightarrow \quad 30\left[\frac{x+2}{5} + \frac{x-1}{6}\right] = \left(\frac{3}{5}\right)30 \quad \Longrightarrow$

$\Longrightarrow \quad 6(x+2) + 5(x-1) = 3 \cdot 6 \quad \Longrightarrow \quad 6x + 12 + 5x - 5 = 18 \quad \Longrightarrow \quad 11x = 11 \quad \Longrightarrow$

$\Longrightarrow \quad x = 1.$

Check: $\frac{(1)+2}{5} + \frac{(1)-1}{6} \stackrel{?}{=} \frac{3}{5} \quad \Longrightarrow \quad \frac{3}{5} + 0 = \frac{3}{5}.$ ✓

19. $\frac{a}{a-5} + \frac{2}{a-6} = \frac{2}{a^2-11a+30} \quad \Longrightarrow \quad \frac{a}{a-5} + \frac{2}{a-6} = \frac{2}{(a-5)(a-6)} \quad \Longrightarrow$

$\Longrightarrow \quad (a-5)(a-6)\left[\frac{a}{a-5} + \frac{2}{a-6}\right] = \left[\frac{2}{(a-5)(a-6)}\right](a-5)(a-6) \quad \Longrightarrow$

$\Longrightarrow \quad a(a-6) + 2(a-5) = 2 \quad \Longrightarrow \quad a^2 - 6a + 2a - 10 = 2 \quad \Longrightarrow$

$\Longrightarrow \quad a^2 - 4a - 12 = 0 \quad \Longrightarrow \quad a^2 - 6a + 2a - 12 = 0 \quad \Longrightarrow$

$\Longrightarrow \quad a(a-6) + 2(a-6) = 0 \quad \Longrightarrow \quad (a-6)(a+2) = 0 \quad \Longrightarrow \quad \cancel{a = 6}, a = -2.$

Check: $\frac{(-2)}{(-2)-5} + \frac{2}{(-2)-6} \stackrel{?}{=} \frac{2}{(-2)^2-11(-2)+30} \quad \Longrightarrow \quad \frac{(-2)}{-7} + \frac{2}{-8} \stackrel{?}{=} \frac{2}{4+22+30} \quad \Longrightarrow$

$\Longrightarrow \quad \frac{16}{56} + \frac{-14}{56} \stackrel{?}{=} \frac{2}{56} \quad \Longrightarrow \quad \frac{2}{56} = \frac{2}{56}.$ ✓