

The Weekly Rigor

No. 364

“A mathematician is a machine for turning coffee into theorems.”

June 12, 2021

50 Word Problems Involving Rational Equations (Part 14)

SELECTED SOLUTIONS

6. $\frac{x}{x+3} \cdot \frac{x+7}{x+10} = \frac{3}{4} \Rightarrow 4(x+7) = 3(x+10) \Rightarrow 4x+28 = 3x+30 \Rightarrow x=2.$
 $\therefore$ The original fraction is $\frac{2}{5}.$

7. Let the two positive integers be x and $x+4$. $\frac{1}{x} + \frac{1}{x+4} = \frac{10}{21} \Rightarrow x=3.$
 $\therefore$ The two positive integers are 3 and 7.

11.

	Distance	Rate	Time
Wendy	30	$x+5$	$\frac{30}{x+5}$
Kim	20	x	$\frac{20}{x}$

$\frac{30}{x+5} = \frac{20}{x} \Rightarrow 30x = 20(x+5) \Rightarrow x=10. \therefore$ Kim rides at 10 mph and Wendy 15.

13.

	Distance	Rate	Time
Upstream	12	$x-3$	$\frac{12}{x-3}$
Downstream	24	$x+3$	$\frac{24}{x+3}$

Let x be the speed in still water.

$\frac{12}{x-3} = \frac{24}{x+3} \Rightarrow 12(x+3) = 24(x-3) \Rightarrow x=9. \therefore$ The speed of the boat in still water is 9 mph.

17.

	Distance	Rate	Time
Freight	180	x	$\frac{180}{x}$
Express	195	$x + 20$	$\frac{195}{x + 20}$

Note that $\frac{180}{x} > \frac{195}{x+20}$.

$$\frac{180}{x} = \frac{195}{x+20} + 1 \Rightarrow 180(x+20) = 195x + x(x+20) \Rightarrow x = 45.$$

$\therefore$ The freight train goes at 45 mph and takes $\frac{180}{45} = 4$ hours. The express train goes 65 mph and takes $\frac{195}{65} = 3$ hours.

18.

	Distance	Rate	Time
Sue	60	$x + 10$	$\frac{60}{x+10}$
Doreen	50	x	$\frac{50}{x}$

Note that $\frac{50}{x} > \frac{60}{x+10}$.

$$\frac{60}{x+10} + 2 = \frac{50}{x} \Rightarrow 60x + 2x(x+10) = 50x(x+10) \Rightarrow x = 10.$$

$\therefore$ Doreen goes at 10 miles per hour; Sue goes at 20. Doreen takes $\frac{50}{10} = 5$ hours; Sue takes $\frac{60}{10+10} = \frac{60}{20} = 3$ hours.

“Only he who never plays, never loses.”

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