The Weekly Rigor

No. 365 "A mathematician is a machine for turning coffee into theorems." June 19, 2021

50 Word Problems Involving Rational Equations

(Part 15)

21.

	Distance	Rate	Time
1st Car	700	x	$\frac{700}{x}$
2nd Car	700	$x-30$	$\frac{700}{x-30}$

Note that $\frac{700}{x} < \frac{700}{x-30}$.

$\frac{700}{x} + 3 = \frac{700}{x-30} \quad \Longrightarrow \quad x^2 - 30x - 7000 = 0 \quad \Longrightarrow \quad x = 100.$

∴ The 1st Car goes at 100 km/h. The 2nd Car goes at 70 km/h.

22.

	Distance	Rate	Time
John	330	x	$\frac{330}{x}$
Bob	330	$x+5$	$\frac{330}{x+5}$

☞ Bob takes more time than he otherwise would have, due to the flat tire.

Note that $\frac{330}{x} > \frac{330}{x+5}$. Furthermore, $30 \text{ minutes} = \frac{1}{2}\text{hour}$.

$\frac{330}{x} = \frac{330}{x+5} + \frac{1}{2} \quad \Longrightarrow \quad x^2 + 5x - 3300 = 0 \quad \Longrightarrow \quad x = 55.$

∴ John's speed is 55 km/h. Bob's speed is 60 km/h.

23.

	Distance	Rate	Time
Run	2000	x	$\frac{2000}{x}$
Bike	2000	$x+4$	$\frac{2000}{x+4}$

Note that $\frac{2000}{x+4} < \frac{2000}{x}$. Furthermore, 3 minutes 20 seconds $= 3 \times 60 + 20 = 200$ seconds.

$\frac{2000}{x+4} + 200 = \frac{2000}{x} \quad \Longrightarrow \quad x^2 + 4x - 40 = 0 \quad \Longrightarrow \quad x \cong 4.6.$

$\therefore$ David runs at about 4.6 m/s.

24.

	Distance	Rate	Time
Laney	$80x$	80	x
Drew	$100(x-1)$	100	$x-1$

☞ Note the phrase "one hour later," and hence Drew's time of $x-1$ if x is Laney's time.

$80x = 100(x-1) \quad \Longrightarrow \quad x = 5. \ \therefore \ 80(5) = 400$ km.

25.

	Distance	Rate	Time
Car A (slower)	450	x	$\frac{450}{x}$
Car B	600	$x+30$	$\frac{600}{x+30}$

☞ "While" = in the same time.

$\frac{450}{x} = \frac{600}{x+30} \quad \Longrightarrow \quad 450(x+30) = 600x \quad \Longrightarrow \quad x = 90.$

$\therefore$ Car A travels at 90 km/h. Car B travels at 120 km/h.