The Weekly Rigor

No. 375 "A mathematician is a machine for turning coffee into theorems." August 28, 2021

Verifying Trigonometric Identities with Simple Arguments Involving the Product of Two Trigonometric Functions: Problems with Solutions

(Part 9)

17. $\sin\theta\cdot\tan\theta=\sin\theta\cdot\frac{\sin\theta}{\cos\theta}=\frac{\sin^2\theta}{\cos\theta}=\frac{1-\cos^2\theta}{\cos\theta}=\frac{1}{\cos\theta}-\cos\theta=\sec\theta-\cos\theta.$

18. $\cos\theta\cdot\cot\theta=\cos\theta\cdot\frac{\cos\theta}{\sin\theta}=\frac{\cos^2\theta}{\sin\theta}=\frac{1-\sin^2\theta}{\sin\theta}=\frac{1}{\sin\theta}-\sin\theta=\csc\theta-\sin\theta.$

19. $\sin\theta\cdot\cos\theta=\sin\theta\cdot\frac{1}{\sec\theta}=\frac{\sin\theta}{\sec\theta}.$

20. $\cos\theta\cdot\sin\theta=\cos\theta\cdot\frac{1}{\csc\theta}=\frac{\cos\theta}{\csc\theta}.$

21. $\sin\theta\cdot\csc\theta=\sin\theta\cdot\frac{1}{\sin\theta}=\frac{\sin\theta}{\sin\theta}=1=\frac{\cos\theta}{\cos\theta}=\cos\theta\cdot\frac{1}{\cos\theta}=\cos\theta\cdot\sec\theta.$

22. $\sin\theta\cdot\csc\theta=\sin\theta\cdot\frac{1}{\sin\theta}=\frac{\sin\theta}{\sin\theta}=1=\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}=\frac{\sin\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}=\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}=\tan\theta\cdot\cot\theta.$

23. $\cos\theta\cdot\sec\theta=\cos\theta\cdot\frac{1}{\cos\theta}=\frac{\cos\theta}{\cos\theta}=1=\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}=\frac{\sin\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}=\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}=\tan\theta\cdot\cot\theta.$

24. $\sin\theta\cdot\csc\theta=\sin\theta\cdot\frac{1}{\sin\theta}=\frac{\sin\theta}{\sin\theta}=1=\sin^2\theta+\cos^2\theta=1\cdot\sin^2\theta+1\cdot\cos^2\theta=$
$=\frac{\cos^2\theta}{\cos^2\theta}\cdot\sin^2\theta+\frac{\sin^2\theta}{\sin^2\theta}\cdot\cos^2\theta=\cos^2\theta\cdot\frac{\sin^2\theta}{\cos^2\theta}+\sin^2\theta\cdot\frac{\cos^2\theta}{\sin^2\theta}=$
$=\cos^2\theta\cdot\tan^2\theta+\sin^2\theta\cdot\cot^2\theta.$

25. $\cos\theta\cdot\sec\theta=\cos\theta\cdot\frac{1}{\cos\theta}=\frac{\cos\theta}{\cos\theta}=1=\sin^2\theta+\cos^2\theta=1\cdot\sin^2\theta+1\cdot\cos^2\theta=$
$=\frac{\cos^2\theta}{\cos^2\theta}\cdot\sin^2\theta+\frac{\sin^2\theta}{\sin^2\theta}\cdot\cos^2\theta=\cos^2\theta\cdot\frac{\sin^2\theta}{\cos^2\theta}+\sin^2\theta\cdot\frac{\cos^2\theta}{\sin^2\theta}=$
$=\cos^2\theta\cdot\tan^2\theta+\sin^2\theta\cdot\cot^2\theta.$

26. $\tan\theta\cdot\cot\theta=\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}=1=\sin^{\ 2}\theta+\cos^{\ 2}\theta=1\cdot\sin^{\ 2}\theta+1\cdot\cos^{\ 2}\theta=$
$=\frac{\cos^{\ 2}\theta}{\cos^{\ 2}\theta}\cdot\sin^{\ 2}\theta+\frac{\sin^{\ 2}\theta}{\sin^{\ 2}\theta}\cdot\cos^{\ 2}\theta=\cos^{\ 2}\theta\cdot\frac{\sin^{\ 2}\theta}{\cos^{\ 2}\theta}+\sin^{\ 2}\theta\cdot\frac{\cos^{\ 2}\theta}{\sin^{\ 2}\theta}=$
$=\cos^{\ 2}\theta\cdot\tan^{\ 2}\theta+\sin^{\ 2}\theta\cdot\cot^{\ 2}\theta.$

27. $\tan\theta\cdot\sec\theta=\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}=\frac{\sin\theta}{\cos^{\ 2}\theta}=\frac{\sin\theta}{1-\sin^{\ 2}\theta}=\frac{1\cdot\sin\theta}{1-1\cdot\sin^{\ 2}\theta}=\frac{\frac{\cos\theta}{\cos\theta}\cdot\sin\theta}{1-\frac{\cos^{\ 2}\theta}{\cos^{\ 2}\theta}\cdot\sin^{\ 2}\theta}=$

$=\frac{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}}{1-\cos^{\ 2}\theta\cdot\frac{\sin^{\ 2}\theta}{\cos^{\ 2}\theta}}=\frac{\cos\theta\cdot\tan\theta}{1-\cos^{\ 2}\theta\cdot\tan^{\ 2}\theta}.$

28. $\cot\theta\cdot\csc\theta=\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}=\frac{\cos\theta}{\sin^{\ 2}\theta}=\frac{\cos\theta}{1-\cos^{\ 2}\theta}=\frac{1\cdot\cos\theta}{1-1\cdot\cos^{\ 2}\theta}=\frac{\frac{\sin\theta}{\sin\theta}\cdot\cos\theta}{1-\frac{\sin^{\ 2}\theta}{\sin^{\ 2}\theta}\cdot\cos^{\ 2}\theta}=$

$=\frac{\sin\theta\cdot\frac{\cos\theta}{\sin\theta}}{1-\sin^{\ 2}\theta\cdot\frac{\cos^{\ 2}\theta}{\sin^{\ 2}\theta}}=\frac{\sin\theta\cdot\cot\theta}{1-\sin^{\ 2}\theta\cdot\cot^{\ 2}\theta}.$

29. $\sin\theta\cdot\tan\theta=\sin\theta\cdot\frac{\sin\theta}{\cos\theta}=\frac{\sin^{\ 2}\theta}{\cos\theta}=\frac{1-\cos^{\ 2}\theta}{\cos\theta}=\frac{1}{\cos\theta}-\cos\theta=1\cdot\frac{1}{\cos\theta}-1\cdot\cos\theta=$
$=\frac{\sin\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}-\frac{\sin\theta}{\sin\theta}\cdot\cos\theta=\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}-\sin\theta\cdot\frac{\cos\theta}{\sin\theta}=\tan\theta\cdot\csc\theta-\sin\theta\cdot\cot\theta.$

30. $\cos\theta\cdot\cot\theta=\cos\theta\cdot\frac{\cos\theta}{\sin\theta}=\frac{\cos^{\ 2}\theta}{\sin\theta}=\frac{1-\sin^{\ 2}\theta}{\sin\theta}=\frac{1}{\sin\theta}-\sin\theta=1\cdot\frac{1}{\sin\theta}-1\cdot\sin\theta=$
$=\frac{\cos\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}-\frac{\cos\theta}{\cos\theta}\cdot\sin\theta=\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}-\cos\theta\cdot\frac{\sin\theta}{\cos\theta}=\cot\theta\cdot\sec\theta-\cos\theta\cdot\tan\theta.$

31. $\sin\theta\cdot\cos\theta=\sin\theta\cdot\frac{1}{\sec\theta}=\frac{\sin\theta}{\sec\theta}=\frac{\sin\theta}{\left(\frac{1}{\cos\theta}\right)}=\frac{1\cdot\sin\theta}{1\cdot\left(\frac{1}{\cos\theta}\right)}=\frac{\frac{\cos\theta}{\cos\theta}\cdot\sin\theta}{\frac{\sin\theta}{\sin\theta}\cdot\left(\frac{1}{\cos\theta}\right)}=\frac{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}}=$
$=\frac{\cos\theta\cdot\tan\theta}{\tan\theta\cdot\csc\theta}.$

32. $\cos\theta\cdot\sin\theta=\cos\theta\cdot\frac{1}{\csc\theta}=\frac{\cos\theta}{\csc\theta}=\frac{\cos\theta}{\left(\frac{1}{\sin\theta}\right)}=\frac{1\cdot\cos\theta}{1\cdot\left(\frac{1}{\sin\theta}\right)}=\frac{\frac{\sin\theta}{\sin\theta}\cdot\cos\theta}{\frac{\cos\theta}{\cos\theta}\cdot\left(\frac{1}{\sin\theta}\right)}=\frac{\sin\theta\cdot\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}}=$
$=\frac{\sin\theta\cdot\cot\theta}{\cot\theta\cdot\sec\theta}.$