

The Weekly Rigor

No. 399

“A mathematician is a machine for turning coffee into theorems.”

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Verifying Trigonometric Identities with Simple Arguments Involving the Product of Three Trigonometric Functions: Problems with Solutions (Part 24)

97. $\tan \theta \cdot \csc \theta \cdot \csc \theta = \cot \theta \cdot \sec \theta \cdot \sec \theta.$

SOLUTIONS

1. $\sin \theta \cdot \sin \theta \cdot \csc \theta = \sin \theta \cdot \sin \theta \cdot \frac{1}{\sin \theta} = \sin \theta \cdot \frac{\sin \theta}{\sin \theta} = \sin \theta \cdot 1 = \sin \theta.$

2. $\cos \theta \cdot \cos \theta \cdot \sec \theta = \cos \theta \cdot \cos \theta \cdot \frac{1}{\cos \theta} = \cos \theta \cdot \frac{\cos \theta}{\cos \theta} = \cos \theta \cdot 1 = \cos \theta.$

3. $\sin \theta \cdot \cos \theta \cdot \sec \theta = \sin \theta \cdot \cos \theta \cdot \frac{1}{\cos \theta} = \sin \theta \cdot \frac{\cos \theta}{\cos \theta} = \sin \theta \cdot 1 = \sin \theta.$

4. $\cos \theta \cdot \sin \theta \cdot \csc \theta = \cos \theta \cdot \sin \theta \cdot \frac{1}{\sin \theta} = \cos \theta \cdot \frac{\sin \theta}{\sin \theta} = \cos \theta \cdot 1 = \cos \theta.$

5. $\sin \theta \cdot \tan \theta \cdot \csc \theta = \sin \theta \cdot \tan \theta \cdot \frac{1}{\sin \theta} = \tan \theta \cdot \frac{\sin \theta}{\sin \theta} = \tan \theta \cdot 1 = \tan \theta.$

6. $\cos \theta \cdot \cot \theta \cdot \sec \theta = \cos \theta \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{\cos \theta}{\sin \theta} \cdot 1 = \frac{\cos \theta}{\sin \theta} = \cot \theta.$
7. $\sin \theta \cdot \tan \theta \cdot \cot \theta = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \sin \theta \cdot \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \sin \theta \cdot 1 \cdot 1 = \sin \theta.$
8. $\cos \theta \cdot \cot \theta \cdot \tan \theta = \cos \theta \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \cos \theta \cdot \frac{\cos \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} = \cos \theta \cdot 1 \cdot 1 = \cos \theta.$
9. $\sin \theta \cdot \sec \theta \cdot \csc \theta = \sin \theta \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{1}{\cos \theta} \cdot 1 = \frac{1}{\cos \theta} = \sec \theta.$
10. $\cos \theta \cdot \csc \theta \cdot \sec \theta = \cos \theta \cdot \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} = \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{1}{\sin \theta} \cdot 1 = \frac{1}{\sin \theta} = \csc \theta.$
11. $\sin \theta \cdot \csc \theta \cdot \csc \theta = \sin \theta \cdot \frac{1}{\sin \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{1}{\sin \theta} \cdot 1 = \frac{1}{\sin \theta} = \csc \theta.$
12. $\cos \theta \cdot \sec \theta \cdot \sec \theta = \cos \theta \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\cos \theta} = \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{1}{\cos \theta} \cdot 1 = \frac{1}{\cos \theta} = \sec \theta.$
13. $\sin \theta \cdot \csc \theta \cdot \cot \theta = \sin \theta \cdot \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot 1 = \frac{\cos \theta}{\sin \theta} = \cot \theta.$
14. $\cos \theta \cdot \sec \theta \cdot \tan \theta = \cos \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot 1 = \frac{\sin \theta}{\cos \theta} = \tan \theta.$
15. $\tan \theta \cdot \csc \theta \cdot \cot \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{1}{\sin \theta} \cdot 1 \cdot 1 = \frac{1}{\sin \theta} = \csc \theta.$
16. $\cot \theta \cdot \sec \theta \cdot \tan \theta = \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{1}{\cos \theta} \cdot 1 \cdot 1 = \frac{1}{\cos \theta} = \sec \theta.$
17. $\tan \theta \cdot \tan \theta \cdot \cot \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot 1 \cdot 1 = \frac{\sin \theta}{\cos \theta} = \tan \theta.$
18. $\cot \theta \cdot \cot \theta \cdot \tan \theta = \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot 1 \cdot 1 = \frac{\cos \theta}{\sin \theta} = \cot \theta.$

“Only he who never plays, never loses.”