The Weekly Rigor

No. 401 "A mathematician is a machine for turning coffee into theorems." February 26, 2022

Verifying Trigonometric Identities with Simple Arguments Involving the Product of Three Trigonometric Functions: Problems with Solutions

(Part 26)

47. $\sin^2\theta \cdot \cos\theta = (1-\cos^2\theta)\cdot\cos\theta = \cos\theta - \cos^3\theta.$

48. $\cos^2\theta \cdot \sin\theta = (1-\sin^2\theta)\cdot\sin\theta = \sin\theta - \sin^3\theta.$

49. $\sin^2\theta \cdot \tan\theta = (1-\cos^2\theta)\cdot\tan\theta = \tan\theta - \cos^2\theta\cdot\tan\theta = \tan\theta - \cos^2\theta\cdot\frac{\sin\theta}{\cos\theta} =$
$= \tan\theta - \cos\theta\cdot\sin\theta.$

50. $\cos^2\theta \cdot \cot\theta = (1-\sin^2\theta)\cdot\cot\theta = \cot\theta - \sin^2\theta\cdot\cot\theta = \cot\theta - \sin^2\theta\cdot\frac{\cos\theta}{\sin\theta} =$
$= \cot\theta - \sin\theta\cdot\cos\theta.$

51. $\sin^2\theta \cdot \sec\theta = (1-\cos^2\theta)\cdot\sec\theta = \sec\theta - \cos^2\theta\cdot\sec\theta = \sec\theta - \cos^2\theta\cdot\frac{1}{\cos\theta} =$
$= \sec\theta - \cos\theta.$

52. $\cos^2\theta \cdot \csc\theta = (1-\sin^2\theta)\cdot\csc\theta = \csc\theta - \sin^2\theta\cdot\csc\theta = \csc\theta - \sin^2\theta\cdot\frac{1}{\sin\theta} =$
$= \csc\theta - \sin\theta.$

53. $\sin^2\theta \cdot \csc\theta = (1-\cos^2\theta)\cdot\csc\theta = \csc\theta - \cos^2\theta\cdot\csc\theta = \csc\theta - \cos^2\theta\cdot\frac{1}{\sin\theta} =$
$= \csc\theta - \cos\theta\cdot\frac{\cos\theta}{\sin\theta} = \csc\theta - \cos\theta\cdot\cot\theta.$

54. $\cos^2\theta \cdot \sec\theta = (1-\sin^2\theta)\cdot\sec\theta = \sec\theta - \sin^2\theta\cdot\sec\theta = \sec\theta - \sin^2\theta\cdot\frac{1}{\cos\theta} =$
$= \sec\theta - \sin\theta\cdot\frac{\sin\theta}{\cos\theta} = \sec\theta - \sin\theta\cdot\tan\theta.$

55. $\sin^2\theta \cdot \cot\theta = (1-\cos^2\theta)\cdot\cot\theta = \cot\theta - \cos^2\theta\cdot\cot\theta.$

56. $\cos^2\theta \cdot \tan\theta = (1-\sin^2\theta)\cdot\tan\theta = \tan\theta - \sin^2\theta\cdot\tan\theta.$

57. $\tan^2\theta \cdot \sin\theta = (\sec^2\theta - 1) \cdot \sin\theta = \sin\theta \cdot \sec^2\theta - \sin\theta = \sin\theta \cdot \frac{1}{\cos^2\theta} - \sin\theta =$
$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} - \sin\theta = \tan\theta \cdot \sec\theta - \sin\theta.$

58. $\cot^2\theta \cdot \cos\theta = (\csc^2\theta - 1) \cdot \cos\theta = \cos\theta \cdot \csc^2\theta - \cos\theta = \cos\theta \cdot \frac{1}{\sin^2\theta} - \cos\theta =$
$= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} - \cos\theta = \cot\theta \cdot \csc\theta - \cos\theta.$

59. $\tan^2\theta \cdot \cos\theta = (\sec^2\theta - 1) \cdot \cos\theta = \cos\theta \cdot \sec^2\theta - \cos\theta = \cos\theta \cdot \frac{1}{\cos^2\theta} - \cos\theta =$
$= \frac{1}{\cos\theta} - \cos\theta = \sec\theta - \cos\theta.$

60. $\cot^2\theta \cdot \sin\theta = (\csc^2\theta - 1) \cdot \sin\theta = \sin\theta \cdot \csc^2\theta - \sin\theta = \sin\theta \cdot \frac{1}{\sin^2\theta} - \sin\theta =$
$= \frac{1}{\sin\theta} - \sin\theta = \csc\theta - \sin\theta.$

61. $\tan^2\theta \cdot \sec\theta = (\sec^2\theta - 1) \cdot \sec\theta = \sec^3\theta - \sec\theta.$

62. $\cot^2\theta \cdot \csc\theta = (\csc^2\theta - 1) \cdot \csc\theta = \csc^3\theta - \csc\theta.$

63. $\tan^2\theta \cdot \csc\theta = (\sec^2\theta - 1) \cdot \csc\theta = \csc\theta \cdot \sec^2\theta - \csc\theta.$

64. $\cot^2\theta \cdot \sec\theta = (\csc^2\theta - 1) \cdot \sec\theta = \sec\theta \cdot \csc^2\theta - \sec\theta.$

65. $\tan^2\theta \cdot \cot\theta = (\sec^2\theta - 1) \cdot \cot\theta = \cot\theta \cdot \sec^2\theta - \cot\theta = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos^2\theta} - \cot\theta =$
$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} - \cot\theta = \csc\theta \cdot \sec\theta - \cot\theta.$

66. $\cot^2\theta \cdot \tan\theta = (\csc^2\theta - 1) \cdot \tan\theta = \tan\theta \cdot \csc^2\theta - \tan\theta = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin^2\theta} - \tan\theta =$
$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} - \tan\theta = \sec\theta \cdot \csc\theta - \tan\theta.$