The Weekly Rigor

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Verifying Trigonometric Identities with Simple Arguments Involving the Product of Three Trigonometric Functions: Problems with Solutions

(Part 27)

67. $\sec^2\theta \cdot \sin\theta = \left(1+\tan^2\theta\right)\cdot\sin\theta = \sin\theta + \tan^2\theta\cdot\sin\theta = \sin\theta + \frac{\sin^2\theta}{\cos^2\theta}\cdot\sin\theta =$
$= \frac{\cos^2\theta\cdot\sin\theta+\sin^3\theta}{\cos^2\theta} = \frac{\sin\theta\left(\cos^2\theta+\sin^2\theta\right)}{\cos^2\theta} = \frac{\sin\theta\cdot 1}{\cos^2\theta} = \frac{\sin\theta}{\cos^2\theta} = \frac{\sin\theta}{1-\sin^2\theta}.$

Alternate verification:
$\sec^2\theta\cdot\sin\theta = \frac{1}{\cos^2\theta}\cdot\frac{\sin\theta}{1} = \frac{\sin\theta}{\cos^2\theta} = \frac{\sin\theta}{1-\sin^2\theta}.$

68. $\csc^2\theta \cdot \cos\theta = \left(1+\cot^2\theta\right)\cdot\cos\theta = \cos\theta + \cot^2\theta\cdot\cos\theta = \cos\theta + \frac{\cos^2\theta}{\sin^2\theta}\cdot\cos\theta =$
$= \frac{\sin^2\theta\cdot\cos\theta+\cos^3\theta}{\sin^2\theta} = \frac{\cos\theta\left(\sin^2\theta+\cos^2\theta\right)}{\sin^2\theta} = \frac{\cos\theta\cdot 1}{\sin^2\theta} = \frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-\cos^2\theta}.$

Alternate verification:
$\csc^2\theta\cdot\cos\theta = \frac{1}{\sin^2\theta}\cdot\frac{\cos\theta}{1} = \frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-\cos^2\theta}.$

69. $\sec^2\theta\cdot\cos\theta = \left(1+\tan^2\theta\right)\cdot\cos\theta = \cos\theta + \cos\theta\cdot\tan^2\theta.$

70. $\csc^2\theta\cdot\sin\theta = \left(1+\cot^2\theta\right)\cdot\sin\theta = \sin\theta + \sin\theta\cdot\cot^2\theta.$

71. $\sec^2\theta\cdot\tan\theta = \left(1+\tan^2\theta\right)\cdot\tan\theta = \tan\theta + \tan^3\theta.$

72. $\csc^2\theta\cdot\cot\theta = \left(1+\cot^2\theta\right)\cdot\cot\theta = \cot\theta + \cot^3\theta.$

73. $\sec^2\theta\cdot\csc\theta = \left(1+\tan^2\theta\right)\cdot\csc\theta = \csc\theta + \csc\theta\cdot\tan^2\theta.$

74. $\csc^2\theta\cdot\sec\theta = \left(1+\cot^2\theta\right)\cdot\sec\theta = \sec\theta + \sec\theta\cdot\cot^2\theta.$

75. $\sec^2\theta \cdot \cot\theta = \left(1 + \tan^2\theta\right) \cdot \cot\theta = \cot\theta + \tan^2\theta \cdot \cot\theta = \cot\theta + \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta} =$
$= \cot\theta + \frac{\sin^2\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos^2\theta} = \cot\theta + \frac{\sin\theta}{\cos\theta} = \cot\theta + \tan\theta.$

76. $\csc^2\theta \cdot \tan\theta = \left(1 + \cot^2\theta\right) \cdot \tan\theta = \tan\theta + \cot^2\theta \cdot \tan\theta = \tan\theta + \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\sin\theta}{\cos\theta} =$
$= \tan\theta + \frac{\cos^2\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin^2\theta} = \tan\theta + \frac{\cos\theta}{\sin\theta} = \tan\theta + \cot\theta.$

77. $\sin\theta \cdot \sin\theta \cdot \csc\theta = \sin\theta \cdot \sin\theta \cdot \frac{1}{\sin\theta} = \sin\theta \cdot \frac{\sin\theta}{\sin\theta} = \sin\theta \cdot 1 = \sin\theta \cdot \frac{\cos\theta}{\cos\theta} =$
$= \sin\theta \cdot \cos\theta \cdot \frac{1}{\cos\theta} = \sin\theta \cdot \cos\theta \cdot \sec\theta.$

78. $\sin\theta \cdot \sin\theta \cdot \csc\theta = \sin\theta \cdot \sin\theta \cdot \frac{1}{\sin\theta} = \sin\theta \cdot \frac{\sin\theta}{\sin\theta} = \sin\theta \cdot 1 \cdot 1 = \sin\theta \cdot \frac{\sin\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} =$
$= \sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \sin\theta \cdot \tan\theta \cdot \cot\theta.$

79. $\sin\theta \cdot \cos\theta \cdot \sec\theta = \sin\theta \cdot \cos\theta \cdot \frac{1}{\cos\theta} = \sin\theta \cdot \frac{\cos\theta}{\cos\theta} = \sin\theta \cdot 1 \cdot 1 = \sin\theta \cdot \frac{\sin\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} =$
$= \sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \sin\theta \cdot \tan\theta \cdot \cot\theta.$

80. $\cos\theta \cdot \cos\theta \cdot \sec\theta = \cos\theta \cdot \sin\theta \cdot \frac{1}{\sin\theta} = \cos\theta \cdot \frac{\sin\theta}{\sin\theta} = \cos\theta \cdot 1 = \cos\theta \cdot \frac{\sin\theta}{\sin\theta} =$
$= \cos\theta \cdot \sin\theta \cdot \frac{1}{\sin\theta} = \cos\theta \cdot \sin\theta \cdot \csc\theta.$

81. $\cos\theta \cdot \cos\theta \cdot \sec\theta = \cos\theta \cdot \cos\theta \cdot \frac{1}{\cos\theta} = \cos\theta \cdot \frac{\cos\theta}{\cos\theta} = \cos\theta \cdot 1 \cdot 1 =$
$= \cos\theta \cdot \frac{\cos\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} = \cos\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \cos\theta \cdot \cot\theta \cdot \tan\theta.$

82. $\cos\theta \cdot \sin\theta \cdot \csc\theta = \cos\theta \cdot \sin\theta \cdot \frac{1}{\sin\theta} = \cos\theta \cdot \frac{\sin\theta}{\sin\theta} = \cos\theta \cdot 1 \cdot 1 =$
$= \cos\theta \cdot \frac{\cos\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} = \cos\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \cos\theta \cdot \cot\theta \cdot \tan\theta.$

83. $\sin\theta \cdot \tan\theta \cdot \csc\theta = \sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = 1 \cdot \tan\theta = \frac{\cos\theta}{\cos\theta} \cdot \tan\theta =$
$= \cos\theta \cdot \frac{1}{\cos\theta} \cdot \tan\theta = \cos\theta \cdot \sec\theta \cdot \tan\theta.$

84. $\sin\theta \cdot \tan\theta \cdot \csc\theta = \sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} = \tan\theta \cdot \cot\theta \cdot \cot\theta.$