The Weekly Rigor

No. 403 "A mathematician is a machine for turning coffee into theorems." March 12, 2022

Verifying Trigonometric Identities with Simple Arguments Involving the Product of Three Trigonometric Functions: Problems with Solutions

(Part 28)

85. $\cos\theta \cdot \sec\theta \cdot \tan\theta = \cos\theta \cdot \frac{1}{\cos\theta} \cdot \tan\theta = \frac{\cos\theta}{\cos\theta} \cdot \tan\theta = \tan\theta = \tan\theta \cdot 1 \cdot 1 =$
$= \tan\theta \cdot \frac{\sin\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} = \tan\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \tan\theta \cdot \tan\theta \cdot \cot\theta.$

86. $\sin\theta \cdot \sec\theta \cdot \csc\theta = \sin\theta \cdot \sec\theta \cdot \frac{1}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \sec\theta = 1 \cdot \sec\theta = \frac{\cos\theta}{\cos\theta} \cdot \sec\theta =$
$= \cos\theta \cdot \frac{1}{\cos\theta} \cdot \sec\theta = \cos\theta \cdot \sec\theta \cdot \sec\theta.$

87. $\sin\theta \cdot \sec\theta \cdot \csc\theta = \sin\theta \cdot \sec\theta \cdot \frac{1}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \sec\theta = 1 \cdot \sec\theta = 1 \cdot \sec\theta \cdot 1 =$
$= \frac{\cos\theta}{\cos\theta} \cdot \sec\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} \cdot \sec\theta \cdot \frac{\sin\theta}{\cos\theta} = \cot\theta \cdot \sec\theta \cdot \tan\theta.$

88. $\cos\theta \cdot \sec\theta \cdot \sec\theta = \cos\theta \cdot \sec\theta \cdot \frac{1}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \cdot \sec\theta = 1 \cdot \sec\theta = 1 \cdot \sec\theta \cdot 1 =$
$= \frac{\cos\theta}{\cos\theta} \cdot \sec\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} \cdot \sec\theta \cdot \frac{\sin\theta}{\cos\theta} = \cot\theta \cdot \sec\theta \cdot \tan\theta.$

89. $\cos\theta \cdot \csc\theta \cdot \sec\theta = \cos\theta \cdot \csc\theta \cdot \frac{1}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \cdot \csc\theta = 1 \cdot \csc\theta = \frac{\sin\theta}{\sin\theta} \cdot \csc\theta =$
$= \sin\theta \cdot \frac{1}{\sin\theta} \cdot \csc\theta = \sin\theta \cdot \csc\theta \cdot \csc\theta.$

90. $\cos\theta \cdot \csc\theta \cdot \sec\theta = \cos\theta \cdot \csc\theta \cdot \frac{1}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \cdot \csc\theta = 1 \cdot \csc\theta = 1 \cdot \csc\theta \cdot 1 =$
$= \frac{\sin\theta}{\sin\theta} \cdot \csc\theta \cdot \frac{\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta} \cdot \csc\theta \cdot \frac{\cos\theta}{\sin\theta} = \tan\theta \cdot \csc\theta \cdot \cot\theta.$

91. $\sin\theta \cdot \csc\theta \cdot \csc\theta = \sin\theta \cdot \csc\theta \cdot \frac{1}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \csc\theta = 1 \cdot \csc\theta = 1 \cdot \csc\theta \cdot 1 =$
$= \frac{\sin\theta}{\sin\theta} \cdot \csc\theta \cdot \frac{\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta} \cdot \csc\theta \cdot \frac{\cos\theta}{\sin\theta} = \tan\theta \cdot \csc\theta \cdot \cot\theta.$

92. $\cos\theta \cdot \cot\theta \cdot \sec\theta = \cos\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = 1 \cdot \cot\theta = \frac{\sin\theta}{\sin\theta} \cdot \cot\theta =$
$= \sin\theta \cdot \frac{1}{\sin\theta} \cdot \cot\theta = \sin\theta \cdot \csc\theta \cdot \cot\theta.$

93. $\cos\theta \cdot \cot\theta \cdot \sec\theta = \cos\theta \cdot \cot\theta \cdot \frac{1}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \cdot \cot\theta = 1 \cdot \cot\theta = 1 \cdot \cot\theta \cdot 1 =$
$= \frac{\cos\theta}{\cos\theta} \cdot \cot\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} \cdot \cot\theta \cdot \frac{\sin\theta}{\cos\theta} = \cot\theta \cdot \cot\theta \cdot \tan\theta.$

94. $\sin\theta \cdot \csc\theta \cdot \cot\theta = \sin\theta \cdot \frac{1}{\sin\theta} \cdot \cot\theta = \frac{\sin\theta}{\sin\theta} \cdot \cot\theta = 1 \cdot \cot\theta = 1 \cdot \cot\theta \cdot 1 =$
$= \frac{\cos\theta}{\cos\theta} \cdot \cot\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} \cdot \cot\theta \cdot \frac{\sin\theta}{\cos\theta} = \cot\theta \cdot \cot\theta \cdot \tan\theta.$

95. $\sin\theta \cdot \sin\theta \cdot \cot\theta = \sin\theta \cdot \sin\theta \cdot \frac{\cos\theta}{\sin\theta} = \sin\theta \cdot \cos\theta \cdot \frac{\sin\theta}{\sin\theta} = \sin\theta \cdot \cos\theta \cdot 1 =$
$\sin\theta \cdot \cos\theta \cdot \frac{\cos\theta}{\cos\theta} = \cos\theta \cdot \cos\theta \cdot \frac{\sin\theta}{\cos\theta} = \cos\theta \cdot \cos\theta \cdot \tan\theta.$

96. $\sin\theta \cdot \sec\theta \cdot \cot\theta = \sin\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \cos\theta \cdot \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \cos\theta \cdot \csc\theta \cdot \tan\theta.$

97. $\tan\theta \cdot \csc\theta \cdot \csc\theta = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} \cdot \frac{1}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = 1 \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} =$
$= 1 \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{\cos\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\cos\theta} = \cot\theta \cdot \sec\theta \cdot \sec\theta.$