

The Weekly Rigor

No. 403

“A mathematician is a machine for turning coffee into theorems.”

March 12, 2022

Verifying Trigonometric Identities with Simple Arguments Involving the Product of Three Trigonometric Functions: Problems with Solutions (Part 28)

$$\begin{aligned} 85. \quad \cos \theta \cdot \sec \theta \cdot \tan \theta &= \cos \theta \cdot \frac{1}{\cos \theta} \cdot \tan \theta = \frac{\cos \theta}{\cos \theta} \cdot \tan \theta = \tan \theta = \tan \theta \cdot 1 \cdot 1 = \\ &= \tan \theta \cdot \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \tan \theta \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \tan \theta \cdot \tan \theta \cdot \cot \theta. \end{aligned}$$

$$\begin{aligned} 86. \quad \sin \theta \cdot \sec \theta \cdot \csc \theta &= \sin \theta \cdot \sec \theta \cdot \frac{1}{\sin \theta} = \frac{\sin \theta}{\sin \theta} \cdot \sec \theta = 1 \cdot \sec \theta = \frac{\cos \theta}{\cos \theta} \cdot \sec \theta = \\ &= \cos \theta \cdot \frac{1}{\cos \theta} \cdot \sec \theta = \cos \theta \cdot \sec \theta \cdot \sec \theta. \end{aligned}$$

$$\begin{aligned} 87. \quad \sin \theta \cdot \sec \theta \cdot \csc \theta &= \sin \theta \cdot \sec \theta \cdot \frac{1}{\sin \theta} = \frac{\sin \theta}{\sin \theta} \cdot \sec \theta = 1 \cdot \sec \theta = 1 \cdot \sec \theta \cdot 1 = \\ &= \frac{\cos \theta}{\cos \theta} \cdot \sec \theta \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \sec \theta \cdot \frac{\sin \theta}{\cos \theta} = \cot \theta \cdot \sec \theta \cdot \tan \theta. \end{aligned}$$

$$\begin{aligned} 88. \quad \cos \theta \cdot \sec \theta \cdot \sec \theta &= \cos \theta \cdot \sec \theta \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\cos \theta} \cdot \sec \theta = 1 \cdot \sec \theta = 1 \cdot \sec \theta \cdot 1 = \\ &= \frac{\cos \theta}{\cos \theta} \cdot \sec \theta \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \sec \theta \cdot \frac{\sin \theta}{\cos \theta} = \cot \theta \cdot \sec \theta \cdot \tan \theta. \end{aligned}$$

$$\begin{aligned} 89. \quad \cos \theta \cdot \csc \theta \cdot \sec \theta &= \cos \theta \cdot \csc \theta \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\cos \theta} \cdot \csc \theta = 1 \cdot \csc \theta = \frac{\sin \theta}{\sin \theta} \cdot \csc \theta = \\ &= \sin \theta \cdot \frac{1}{\sin \theta} \cdot \csc \theta = \sin \theta \cdot \csc \theta \cdot \csc \theta. \end{aligned}$$

$$\begin{aligned} 90. \quad \cos \theta \cdot \csc \theta \cdot \sec \theta &= \cos \theta \cdot \csc \theta \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\cos \theta} \cdot \csc \theta = 1 \cdot \csc \theta = 1 \cdot \csc \theta \cdot 1 = \\ &= \frac{\sin \theta}{\sin \theta} \cdot \csc \theta \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot \csc \theta \cdot \frac{\cos \theta}{\sin \theta} = \tan \theta \cdot \csc \theta \cdot \cot \theta. \end{aligned}$$

$$\begin{aligned} 91. \quad \sin \theta \cdot \csc \theta \cdot \csc \theta &= \sin \theta \cdot \csc \theta \cdot \frac{1}{\sin \theta} = \frac{\sin \theta}{\sin \theta} \cdot \csc \theta = 1 \cdot \csc \theta = 1 \cdot \csc \theta \cdot 1 = \\ &= \frac{\sin \theta}{\sin \theta} \cdot \csc \theta \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot \csc \theta \cdot \frac{\cos \theta}{\sin \theta} = \tan \theta \cdot \csc \theta \cdot \cot \theta. \end{aligned}$$

$$92. \cos \theta \cdot \cot \theta \cdot \sec \theta = \cos \theta \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = 1 \cdot \cot \theta = \frac{\sin \theta}{\sin \theta} \cdot \cot \theta = \\ = \sin \theta \cdot \frac{1}{\sin \theta} \cdot \cot \theta = \sin \theta \cdot \csc \theta \cdot \cot \theta.$$

$$93. \cos \theta \cdot \cot \theta \cdot \sec \theta = \cos \theta \cdot \cot \theta \cdot \frac{1}{\cos \theta} = \frac{\cos \theta}{\cos \theta} \cdot \cot \theta = 1 \cdot \cot \theta = 1 \cdot \cot \theta \cdot 1 = \\ = \frac{\cos \theta}{\cos \theta} \cdot \cot \theta \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \cot \theta \cdot \frac{\sin \theta}{\cos \theta} = \cot \theta \cdot \cot \theta \cdot \tan \theta.$$

$$94. \sin \theta \cdot \csc \theta \cdot \cot \theta = \sin \theta \cdot \frac{1}{\sin \theta} \cdot \cot \theta = \frac{\sin \theta}{\sin \theta} \cdot \cot \theta = 1 \cdot \cot \theta = 1 \cdot \cot \theta \cdot 1 = \\ = \frac{\cos \theta}{\cos \theta} \cdot \cot \theta \cdot \frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \cot \theta \cdot \frac{\sin \theta}{\cos \theta} = \cot \theta \cdot \cot \theta \cdot \tan \theta.$$

$$95. \sin \theta \cdot \sin \theta \cdot \cot \theta = \sin \theta \cdot \sin \theta \cdot \frac{\cos \theta}{\sin \theta} = \sin \theta \cdot \cos \theta \cdot \frac{\sin \theta}{\sin \theta} = \sin \theta \cdot \cos \theta \cdot 1 = \\ \sin \theta \cdot \cos \theta \cdot \frac{\cos \theta}{\cos \theta} = \cos \theta \cdot \cos \theta \cdot \frac{\sin \theta}{\cos \theta} = \cos \theta \cdot \cos \theta \cdot \tan \theta.$$

$$96. \sin \theta \cdot \sec \theta \cdot \cot \theta = \sin \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta \cdot \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \cos \theta \cdot \csc \theta \cdot \tan \theta.$$

$$97. \tan \theta \cdot \csc \theta \cdot \csc \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \frac{1}{\sin \theta} = \frac{\sin \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = 1 \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \\ = 1 \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{\cos \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\cos \theta} = \cot \theta \cdot \sec \theta \cdot \sec \theta.$$

“Only he who never plays, never loses.”